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Perspectives for Mathematics Education in higher education from research on modeling in differential equations teaching

Perspectivas para a Educação Matemática no ensino superior a partir da pesquisa sobre modelagem no ensino de equações

ABSTRACT

The present article introduces perspectives for investigating Mathematics Education in Higher Education based on qualitative research that analyzed the potential contributions of Mathematical Modeling assignments about population growth for Differential Equations learning. Participants were students in the Mathematics Degree program at a private university in the interior of the Brazilian state of Minas Gerais, enrolled in a Differential Equations course taught remotely because of the Covid-19 pandemic. Based on the results, the Modeling assignments contributed to re-signifying concepts and applications of Ordinary Differential Equations and developing a critical perspective among future Mathematics teachers, mainly about issues such as citizenship, social inequality, and the social role of Mathematics itself. We aimed at seeking future perspectives for investigations about Mathematics Education in Higher Education based on the aforementioned contributions.

Keywords: Mathematics Education in Higher Education. Mathematical Modeling. Educational and Social-critical Perspectives. Differential

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RESUMO

O presente artigo apresenta perspectivas para investigações em Educação Matemática no Ensino Superior, a partir de uma pesquisa qualitativa que analisou potenciais contribuições de atividades de Modelagem Matemática relacionadas ao crescimento populacional para a aprendizagem de Equações Diferenciais. Os participantes foram alunos de um curso de Licenciatura em Matemática de uma universidade particular no interior do estado brasileiro de Minas Gerais, matriculados na disciplina Equações Diferenciais, ministrada remotamente devido à pandemia mundial de Covid-19. Os resultados apontam que as atividades de Modelagem contribuíram para uma ressignificação de conceitos e aplicações de Equações Diferenciais Ordinárias e para o desenvolvimento de uma perspectiva crítica nos futuros professores de Matemática, especialmente, em relação a questões como cidadania, desigualdade social e papel social da própria Matemática. A partir de tais contribuições, buscamos perspectivar futuras investigações em Educação Matemática no Ensino Superior.

Palavras-chave: Educação Matemática no Ensino Superior. Modelagem Matemática. Perspectivas Educacional e Sóciocrítica. Ensino de

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INTRODUCTION

Scientific production in Mathematics Education in Higher Education has been growing and solidifying itself, and this statement can be proven not just by the historical increase in the number of research presented in national and international events, in the Mathematics Education field, but also by the significant number of articles published in journals, as well as by dissertations and theses defended in Post-graduation programs that approach several aspects of teaching and learning processes regarding the Mathematics contents taught in Higher Education, mainly Differential and Integral Calculus, Analytical Geometry, Linear Algebra, Differential Equations, Euclidean Geometry, Algebraic Structures and Real Analysis.

Assumingly, one of the reasons for such a scenario lies on the fact that many performed research point towards an undeniable factor: the high rate of higher education students whose performance in Mathematics learning is below the expectation is a quite concerning issue, as stated by Igliori (2009). According to her, "In our opinion, research plays key role in screening the causes and in identifying the ways to be taken in the search for improvements" (Igliori, 2009, p. 13).

We herein aim at finding and introducing investigation possibilities at Higher Education scope based on a research carried out with students of Mathematics Degree. Its goals were to bring up social reflections to its participants; thus, it is, somehow, different from other research focused on Mathematical Modeling addressed in other training fields.

The main aim of this research was to identify and analyze likely contributions from Mathematical Modeling assignments to Differential Equations (DE) learning in a Mathematics Degree program. These assignments were overall conceived from the educational perspective; mainly from the social-critical perspective, as further shown in this text.

We understand that, based on its theoretical-methodological description, as well as on its conclusions, this research may bring up relevant contributions for investigations on the Mathematics Education in place in Higher Education, mainly for learning and teaching processes focused on Mathematics contents approached in Higher Education. This statement suggests a reflection about the fact that some of its teaching aspects must be revised.

ABOUT DIFFERENTIAL EQUATIONS TEACHING

Yet, there are few research on DE teaching at Mathematics Education scope in Higher Education, in Brazil. Some of them point out that, traditionally, DE studying is more often focused on algebra, although it puts aside its phenomena-modeling potential. Accordingly, and based on our own experience as DE professors, we can trigger some reflections about Modeling as tool to build mathematical concepts and their applications in DE teaching.

We have observed that most students algebraically manipulate their solutions, but they do not actually know how to identify the applications for their solutions. Thus, their focus seems to be limited to the application of solution rules, but without using some mathematical concepts. Therefore, it is possible observing that they are not concerned about Differential and Integral Calculus (Reis, 2001, 2009). Overall, the existing research and teaching practices in Higher Education show that professors "provide ready-to-use formulas", but it does not, necessarily, implies in learning; thus, students perform calculations and finish the solutions.

Oliveira and Igliori (2013) pointed out possibilities or alternatives for DE teaching based on a literature review carried out in Capes database, between 2000 and 2011, which aimed at finding students' difficulties to learn it. Based on the researchers:

> Overall, the selected studies highlighted that Differential Equations teaching has been happening to gather closer attention to analytical solutions based on algebraic manipulations of resolutions and, in such a process, they have reported students' difficulty to lean basic mathematics, to apply derivative and integral concepts, and to interpret instantaneous variation rates. They also highlighted that the very focus that has been given to the content does not help its best understanding, and it can give students a hard time conceiving what a Differential Equation is and, consequently, its application in contextualized problems that demand interpretation. Some studies express students' difficulty in thinking in an algebraic and graphic way, simultaneously (Oliveira & Igliori, 2013, p. 21).

Yet, according to the researchers, most of the assessed authors have pointed out the possibility of DE teaching: "the qualitative profile of this subject, in a contextualized way, based on problem-situations and by using computational resources" (Oliveira & Igliori, 2013, p. 21).

With respect to such a contextualization, we agree with Barbosa (2004), according to whom, knowledge contextualization is an essential tool for the didactic transposition issue; it regards the existing association between teaching and learning processes, and knowledge contextualization, based on their daily experiences and on the meaning of what is learned. This researcher also highlights a much more significant issue concerning DE teaching when he states that knowledge contextualization "implies using contexts that have meaning to students, they must be intellectually and emotionally involved; therefore, it is an essential strategy to build meanings (Barbosa, 2004, p. 41). Based on the aforementioned scenario, we will introduce a Mathematical Modeling proposition that may become an interesting way to build mathematical knowledge within DE, as well as to help its interpretation based on natural-phenomena meanings, with criticality and creativity.

MATHEMATICAL MODELING FROM THE SOCIAL-CRITICAL PERSPECTIVE

Research in the Mathematics' Education field bring along different perspectives of Mathematical Modeling and evidence their outspread in Brazil and abroad, as stated by Araújo and Martins (2017). These researchers state that Modeling can be related to using mathematical knowledge to solve real problems or to solve problems in knowledge fields other than Mathematics; they also place it as trend in Mathematics' Education, which aims at "the performance of assignments in educational contexts, according to which, students are invited to seek solutions for real problems through Mathematics" (Araújo & Martins, 2017, p. 115).

We understand that we can relate the herein reported research to the Mathematical Modeling educational perspective, since it approaches mathematical contents teaching (Kaiser and Sriraman, 2006). Nevertheless, we will also highlight the social-critical perspective for Mathematical Modeling based on Burak (1992), namely: Mathematics teaching methodology substantiated by theoretical references in constructivist and socio-interactionist theories, from a science viewpoint that encompasses knowledge fields other than Mathematics, such as Psychology, Sociology and Anthropology, which are related to Mathematics' Education.

Initially, Burak (1992) proposes a Modeling concept from the Mathematics' Education perspective that seeks to remain closely related to the introduced viewpoint, according to which, Mathematics can be taken as social practice through teaching and learning, since it concerns a community of students, the development of a set of actions that imply the classroom space, as well as a practice guided by principles that involve interests and an anthropological view, and the possibility of building mathematical and interdisciplinary knowledge. Thus, "It is a view that conceives Mathematics as an important instrument, but without disregarding other fields that can be part of Mathematics teaching and learning processes" (Burak, 1992, p. 62).

The researcher argues about Mathematical Modeling from this Mathematics' Education perspective, and suggests that it must "be a set of procedures whose goal is to establish a parallel in order to try to mathematically explain phenomena observed in humans' daily routines to help them making predictions and decisions" (Burak, 1992, p. 62).

Burak (1992) suggests some principles for the Mathematical Modeling practice from this perspective, namely: 1) Starting from the interest of involved individuals (in this case, students); 2) Getting information and data from the environment where the groups' interest is in. Yet, based on Burak and Klüber (2004), these principles.

> Seek to consolidate actions based on the interest of students involved in Modeling assignments. From the socio-constructivist viewpoint, the reason to do something lies on doing something. Interest in the assignment is closely related to intrinsic motivation and it gains strength within the context that nourishes both interest and motivation (Burak & Klüber, 2004, p. 4).

With respect to approaching Mathematical Modeling in classroom, Burak (2010) suggests that its development must be carried out based on the following stages: 1) Choosing the topic; 2) Exploratory research; 3) Screening the problem; 4) Solving the problem and developing the mathematical context; 5) Critical analysis of problem solution.

Next, we will detail the performed research in its own context; we will try to highlight the development of Mathematical Modeling assignments, in light of stages suggested by Burak (2010).

PRESENTING THE RESEARCH IN ITS CONTEXT

The research was carried out in the classroom, with Mathematics' Degree students from a private university located in Ipatinga, Minas Gerais State, who were enrolled in the Ordinary Differential Equations (ODE) course, in the first semester of 2020, and whose professor was the 2nd author of the current article. Assignments in this course were conducted in a remote way due to the Covid-19 pandemic, to the guidelines by the World Health Organization (WHO) and to public bureaus about social distancing. The course was taught in the Google Meet platform, which is institutionally available for all students and professors in the university.

The classes were taught once a week, and last approximately 4 hours. The research was carried out for 4 straight weeks, in June

2020; 12 students were divided into 4 groups, with 3 members each - group formation was spontaneous. They formed WhatsApp groups, as well as groups in other social networks, for communication and discussion activities. Thus, technology became an alternative to be deeply explored by 21st century professors, as argued by Carneiro and Passos (2014):

> The use of technologies in Mathematics classes can lead to changes in classroom dynamics and in the ways of teaching and learning the contents. Therefore, teachers must understand and be clear about the possibilities, and also about technologies' limits (Carneiro & Passos, 2014, p. 1).

Course ODE is mandatory in the course matrix of the program; it accounts for 80h workload. The Mathematics Degree program, in its turn, lasts 8 semesters - the class joining the research was enrolled in the 7th semester.

A lecture about Modeling and Mathematics Education was given as first research assignment; it was lectured during the 1st part of the class, and lasted approximately 2 hours. The following question was made to students in the 2nd part of the class: the conduction of a census by the Brazilian Institute of Geography and Statistics (IBGE) is expected to happen in 2020 to measure the Brazilian population. Is it going to happen?

The students quickly searched on the internet and found that the Census was postponed to 2021 by IBGE, since the pandemic would not allow its conduction. Thus, we triggered the following discussion: Would there be reliable mathematical models to measure the Brazilian population? What would they be?

Then, we asked the students – who were already in their groups – to bring a bibliographic search to the next class related to mathematical models for population prediction, to their validations and limitations.

Thus, we developed the 2 Mathematical Modeling exploratory assignments that have involved variation rate issues and population growth demands, and that were extracted and adapted from Laudares et al. (2017, p. 118-124). It is important highlighting that the first author authorized the use of these activities in our research and that they were essential for us to reach the herein proposed aim. Modeling Assignment 1 concerned the exponential growth of a population variation problem, based on the Law of Malthus. Modeling Assignment 2 regarded a demand analysis and population growth problem, based on the Law of Verhulst.

We added a question to the end of each applied assignment by asking students to make a critical analysis of the Population Variation Issue, based on mathematical laws and on other statistical instruments used in Brazil. Such a questioning aimed at assessing students' criticality about what DE can bring through reflection about the population growth subject and about the public policies in place.

A Questionnaire of Assignments' Evaluation was applied after the exercises were over; it was based on individual virtually-sent answers. We had a discussion with all students about their experience with this work, based on the Mathematical Modeling activity in the last class, after answers were sent back. Thus, data were collected throughout the whole process, through prints of chats in WhatsApp, and in the Google Meet platform, of notes in field journals and of audio and video recordings from all classes – it was authorized by research participants and provided large source for data description and analysis.

INTRODUCING MODELING ASSIGNMENT 2

We herein opted to only introduce the methodological cut of one of the Mathematical Modeling exploratory assignments. We highlight that the full description of the assignments can be found in Araújo (2020).

Modeling Assignment 2 brought along the Demand and Exponential Growth Model Analysis Problem by approaching the Law of Malthus, the Law of Verhulst and the Initial Value Problem (IVP), as follows:

DATA:

(i) Demand and Population Growth Laws

Law of Malthus: the instantaneous variation rate of a population P is proportional to the population found in the considered instant t, i.e., dP/dt=kP whose solution is in the form: P=C.e^(kt) com k >0 as previously seen.

Law of Verhulst: By observing the Law of Malthus and its model equation, which accepts an unlimited population growth (exponential), and by manipulating statistical data from that time, Verhulst observed that a given population, in real life, does not grow in an unlimited way. He found out that, under normal conditions, habitable regions (cities, states and countries) tend to suffer with the effect of population concentration; thus, he assumed that "the relative growth rate of a population linearly decreases if one takes into consideration this population evolution in time". It corresponded to assume K in Malthus' equation as decreasing linear function. It means considering K as population unlimited growth reduction factor triggered by natural resources' scarcity and dispute in real life; Therefore, the logistic model for the Law of Verhulst is: dP/dt= (a-bP)P com a ∈IR e b ∈IR, a >0 e b >0.

(ii) At initial instant t = 0, the population is given by P = P_0.

QUESTIONS:

(iii) Determine the function P = P(t), analytical solution of Verhulst Differential Equation;

(iv) Analyze the conditions to draft the Verhulst model

(v) Draft the Verhulst model's graphics

(vi) Describe in a short text the Malthus and Verhulst's models by comparing the graphics and the equations.

We introduced several didactic steps after interpreting the questions, as follows:

Step 1: 1.1) Identify the variables for solving the problem;

1.2) Mathematically express the Law of Malthus;

1.3) Mathematically express the Law of Verhulst.

Step 2: Describe the initial condition given in the problem.

Step 3: Determine the Verhulst model, which is a 1st-order Differential Equation of the separation of variables type.

Step 4: Analyze the conditions to draft the Verhulst's model.

Step 5: Draft the Verhulst models' graphic.

Step 6: Write a text comparing the graphics and the two laws (Malthus and Verhulst) based on your understanding about the population demand and its growth behavior.

Step 7: Critically analyze the Population Variation Problem based on the Law of Verhulst.

Step 8: Critically analyze the Population Variation Problem based on the Laws of Mathematics and on other statistical instruments used in Brazil.

Now, we will describe solutions and observations in Steps 1 and 2 solved by one of the groups: Group G2.

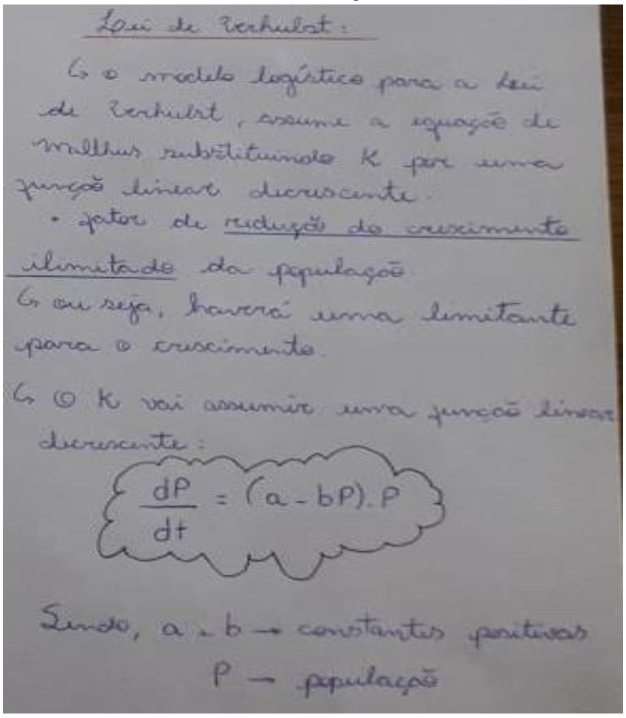
The logistical model for the Law of Verhulst summarizes the Malthus equation by replacing K by a decreasing linear function. K will be population unlimited growth reduction factor, i.e., there will be a growth limit. Thus, the constant K will assume a decreasing function in dP/dt=(a-b.P)P, wherein a and b are positive constants and P is the population, when k=(a-bP).

Group G2 brought in their solution information about the Law of Verhulst by defining it as logistic model, and k assumed the growth reduction factor by defining constants a and b as positive constant and P is represented by the population at any time and the independent variable of the function. This information is inserted in Figure 1, below.

P is the dependent variable and it represents the population at any time, t represents the time and is the independent variable, K is the real constant, dP/dt is the population variation rate. The Law of Malthus is given by dP/dt=k.P , whereas the Law of Verhulst is represented by dP/dt=(a-bP)P; lets observe that the Law of Verhulst is an update of the Law of Malthus (G2).

Our group had a hard time to get to the requested mathematic equation based on the Verhulst's model. We performed in compliance with our understanding, but we do not know if it is correct. The problem, professor, is that the didactic materials available and even the internet do not explain it well, although we learn the concepts of Differential Equations in the classroom. But it is not the same! (G2).

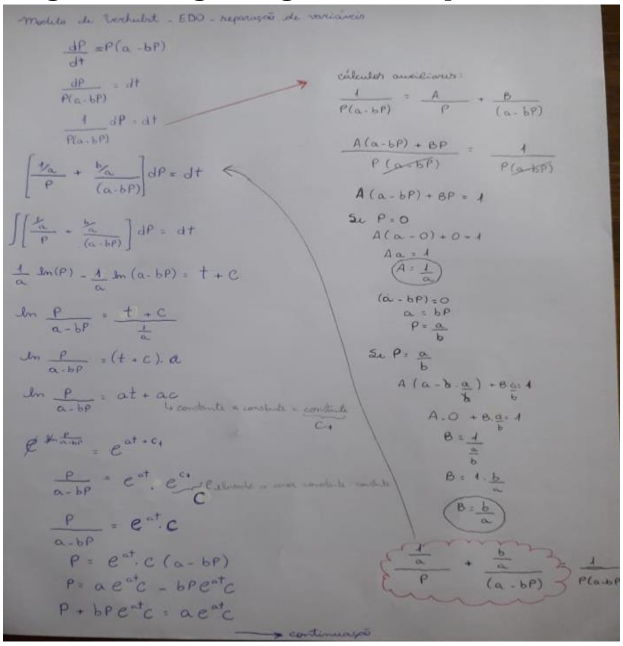
Figure 1 - Interpretation of the Law of Verhulst by G2



Source: Research data.

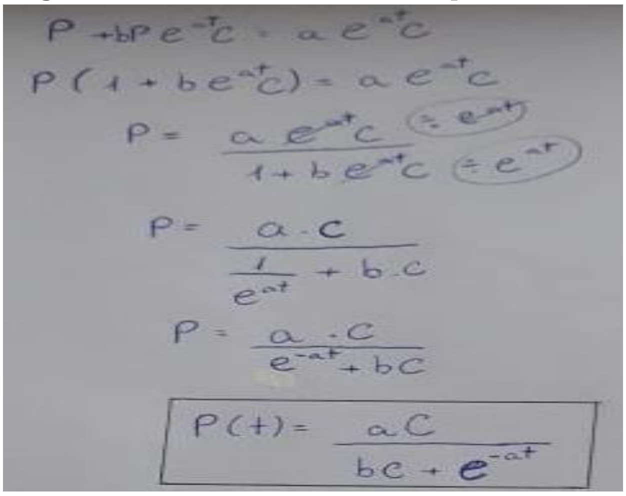
With respect to Step 3, group G2 found an equation defined by P(t)= ac/(bc+ e^(-at)) as shown in Figures 2 and 3.

Figure 2 - Beginning of G2's Step 3 Solution



Source: Research data

Figure 3 - Conclusion of G2's Step 3 Solution



Source: Research data

Below we describe the solution of Verhulst's equation in Charts 1, 2 and 3.

Chart 1 – Decomposing the equation into partial fractions

$$\frac{dP}{dt} = (a - bP).P \;\rightarrow\; \frac{dP}{(a-bP).P} = dt$$

$$\frac{1}{(a-bP).P} = \frac{A}{(a-bP)} + \frac{B}{P} = \frac{A.P+B.(a-bP)}{(a-bP).P}$$ Then, we will have:

$$A.P + B.(a - bP) = 1, \textit{válidos para } P \geq 0$$

$$Se\ P = 0 \;\rightarrow A.0 + B.(a - 0) = 1 \;\rightarrow B = \frac{1}{a}$$

$$Se\ P = \frac{a}{b} \;\rightarrow A.\left(\frac{a}{b}\right) + B.\left[a - b.\left(\frac{a}{b}\right)\right] = 1 \;\rightarrow$$

$$A.\left(\frac{a}{b}\right) + B.(0) = 1 \;\rightarrow A = \frac{b}{a}$$

$$\text{Then: } \frac{dP}{(a-bP).P} = \frac{b}{a.(a-b.P)}dP + \frac{1}{a.P}\,dP = dt$$

Source: Research data

Chart 2 – Integrating the partial fractions

$$I)\ \int \frac{1}{aP}\,dP = \frac{1}{a}.\int \frac{dP}{P} = \frac{1}{a}.\ln|P| + c_1$$

$$II)\ \int \frac{b\,dP}{a.(a-bP)} = \left(\frac{b}{a}\right).\int \frac{dP}{a-bP}$$

Considering: $u = a - b.P$ we will have:

$$\left(\frac{b}{a}\right).\int \frac{(-\,du)}{b.u} = \left(\frac{b}{a}\right).\left(-\frac{1}{b}\right).\int \frac{du}{u} =$$

$$\left(-\frac{1}{a}\right).\ln|a - b.P| + c_2$$

It is so by making the replacement:

$$a - b.P = u \;\rightarrow\; -b.dP = du \;\rightarrow$$

$$dP = \left(-\frac{du}{b}\right)$$

$$\text{III) } \int dt = t + c$$

Then, there will be:

$$\int \frac{b}{a.(a-b.P)}dP + \int \frac{1}{a.P}\,dP = \int dt$$

And it takes us to:

$$\frac{1}{a}\ln|P| - \left(\frac{1}{a}\right).\ln|a - b.P| = t + c_3$$

Source: Research data

Chart 3 – Simplifying the expressions

$$\left(\frac{1}{a}\right).[\ln|P| - \ln|a - b.P|] = t + c \;\Rightarrow$$

$$\left(\frac{1}{a}\right).\ln\left[\frac{|P|}{|a-b.P|}\right] = t + c$$

$$\ln\left[\frac{|P|}{|a-b.P|}\right] = a.(t + c) \;\rightarrow\; \left[\frac{|P|}{|a-b.P|}\right] =$$

$$e^{a.(t+c)} = e^{at}.e^{a.c} = e^{at}.K$$

$$\text{Thus: } |P| = |a - b.P|e^{at}.K$$

We can take out the modules, because we are interested in $\frac{dP}{dt} > 0$ and yet: $0 < P_0 < P$ where, P_0 represents the initial population.

Thus,

$$P = (a - b.P).k.e^{at}$$

At the initial conditions, if t = 0, we will then have: $P = P_0$ and consequently one has:

$$P_0 = (a - b.P_0).k.e^0 = (a - b.P_0).k \;\Rightarrow\; k = \frac{P_0}{(a-b.P_0)}$$

By replacing the value of K in P one finds,

$P = (a - b.P).\left(\frac{P_0}{a-b.P_0}\right).e^{at}$ and it results in:

$$P = \frac{a.P_0 - b.P.P_0}{(a-b.P_0).e^{-at}} = \frac{a.P_0}{(a-b.P_0).e^{-at}} - \frac{b.P.P_0}{(a-b.P_0).e^{-at}}$$

a fact that takes us to:

$$P + \frac{b.P.P_0}{(a-b.P_0).e^{-at}} = \frac{a.P_0}{(a-b.P_0).e^{-at}}$$

$$\frac{P.[(a-b.P_0).e^{-at}] + b.P.P_0}{(a-b.P_0).e^{-at}} = \frac{a.P_0}{(a-b.P_0).e^{-at}}$$

$$P.\left[\frac{(a-b.P_0).e^{-at} + b.P_0}{(a-b.P_0).e^{-at}}\right] = \frac{a.P_0}{(a-b.P_0).e^{-at}}$$

$$P = \frac{a.P_0.[(a-b.P_0).e^{-at}]}{[(a-b.P_0).e^{-at} + b.P_0].(a-b.P_0).e^{-at}}$$

And finally:

$$P = \frac{a.P_0}{b.P_0 + (a-b.P_0).\ e^{-at}} \;\Rightarrow$$

$$P(t) = \frac{a.P_0}{b.P_0 + (a-b.P_0).\ e^{-at}}$$

Source: Research data

Steps 4 to 8 were developed simultaneously, as described below:

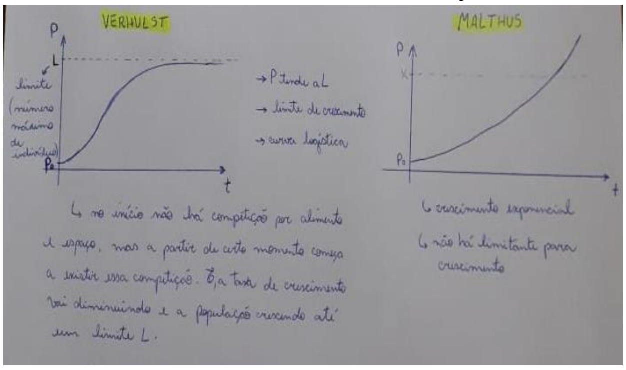
> As we have argued, Malthus' equation does not limit the population due to external factors. Thus, we observed that the Verhulst's equation can be more effective in the calculation of a future population, because it takes into account competition for space, food, among others. At a given longer time t, the Law of Verhulst imposes a growth limit L and is more effective in calculating future populations, as described in the graphic below (G2).

They also argued that the Law of Verhulst is a limiting mathematical model for the Law do Malthus, because both laws are exponential and keep on reinforcing the theory that the population variation rate is proportional to the initial population. Thus, we brought up the following critical argument:

> Actually, we observed that in real life the population does not grow in an unlimited way, since several factors make this chain unstable. Based on our search for this assignment, we realized that the logistical growth in the form of an "S" in the Law of Verhulst is fast, but it loses its speed with time and remains stable when the number of individuals reaches the limit L shown in the graphic below. It happens because of the resources available in the local population environment. This limiting factor of the population happens when factor time "t" tends to a large number and when it corresponds to the study of limits involving the infinite in the Differential and Integral Calculus. We also observed that L in the graphic above is a straight line that is the tangent of the Verhulst curve in the infinite; thus, can it be called asymptote? (G2)

Besides an observation, Group G2 depicted a graphic comparison of the laws of Malthus and Verhulst, as shown in Figure 4.

Figure 4 - Comparison of graphics of the laws of Malthus and Verhulst by G2



Source: Research data

Finally, they analyzed the population variation problem based on mathematical laws and on other statistical instruments used in Brazil; they also provided a summary of their research:

> We were a bit confused given the amount of information in discussions about how to estimate the Brazilian population in 2020. We assessed several websites in the internet, Youtube channels and different mathematical software to calculate the 2020 population projection, which did not meet the current projection presented by IBGE. Our group believes that it happened because the projected polynomial did not take into consideration external factors that have happened (just as previously argued), a fact that also takes place with the Malthus model. The Verhulst model takes into account some limitations, but, yet, it is not perfect. As future Mathematics teachers, we realized that the Mathematical Modeling brought along a message: that teachers must always seek new knowledge, including the non-mathematical ones. We have learned the algebraic part of Differential Equations in the classroom and the present assignment allowed us to understand that it was not enough to have in-depth knowledge about it. By assessing other instruments, we got in contact with other ways of learning and also got to contextualize what we have learned about Differential Equations. We

saw that Mathematics, through its "accounts", brings along numbers that stop being simply numbers and that become reflections, not just for us, Mathematics students, but for the population in general. Our group raised a question, just as an example: In this year of 2020, there will be municipal elections. Why does the rate of women in Brazil is approximately 51.8%, based on the National Research for Continuous Household Sample – IBGE's PNAD (2019 data) – but, nevertheless, the rate of women participation in elective positions in the country does not even reach 15%? (Data provided by Correio Brasiliense, March 08th, 2020) (G2).

We understand that the herein addressed reflections by the students, individually and in their groups, encompassed the Mathematical Modeling stage regarding the "critical analysis of problem solution" proposed by Burak (2010). Thus, we have concluded the trajectory suggested by the researcher by fulfilling them in the assigment concerning Modeling from a social-critical perspective.

CONTRIBUTIONS TO DIFFERENTIAL EQUATIONS LEARNING

We can set some contributions from the Mathematical Modeling assignments from educational and social-critical perspectives for DE learning in the Mathematics Degree program, based on the developed assignment, on records in field journals made throughout the meetings and on answers in the applied questionnaire.

Accordingly, we identified questions concerning learning in participants' subtle speeches, as described below. Collaboration, dialogue and interactions between students and professor-researcher were the main factors for DE teaching and learning processes. This statement was evidenced by the group assignment focused on Modeling, based on the voices of students (who were herein identified by fictional names) and professors, which were extracted from their field journals:

> We got a bit confuse with so much information to deal with in the Mathematical Modeling activities, mainly in Activity 2. However, the dialogue and the group assignment were essential for activities' solution (Luísa, June 2020).
>
> Even after assessing the Differential Equations in classroom, I had many doubts in the Modeling assignment, mainly in the elaboration of the second Law of Verhulst. We made several searches in different contexts, our group had several discussions and we counted on the cooperation of all group members and on the support by the professor, I got to clear many of my doubts about subjects I had not previously understood (Joanis, June 2020).
>
> During the resolution of the 2 blocks of Modeling activities, within the context of the laws of Malthus and Verhults, about Differential Equations, I have observed that there was great interaction and dialogue among participants in every group through social media (Professor, June 2020).

The learning environment enabled by Mathematical Modeling favored DE learning, it boosted multidisciplinary proximity and helped the access to knowledge fields other than the mathematical one, given the fact that Modeling has an open nature:

> We have realized that a positive aspect of Modeling lies on the sample, on how it can be useful in real life, in daily routines, and on how it deals with multi-disciplinary interaction, since it interacts with other knowledge fields. I thought that it was

> interesting how a mathematical model can help solving problems that are not mathematical. I have not though about it, so far. [...] I realized that mathematical Modeling makes us assess and access other means besides didactic books. I also had a hard time solving the assignments, but team work, although virtual, helped a lot (Helena, June 2020).

> I realized that Modeling provides a good environment for learning and it could be used at allteaching levels. I believe that it can be used to develop learning and general skills, because it makes students problematize, investigate, and seek knowledge by themselves, without getting stuck to the professor and to didactic materials (Luísa, June 2020).

> The exploratory research made students seek new knowledge, I realized, as professor-researcher, that the exploratory assignment, just as suggested by Burak, made students assess several dimensions within the population counting context and within a likely mathematical model to predict the Brazilian population by 2020. They wondered, discussed, analyzed and brought up concerns with the mathematical model, despite the fact that results did not report the reality (Professor, June 2020).

The Modeling environment helped DE learning in face of challenges to re-signify different knowledge types:

> I go to understand the Differential Equations concepts based on Modeling. This is the second time that I attend this course and the first time I had a hard time understanding a concept. At this time, I got to understand that we can assess it based on concepts and by associating it with natural phenomena that make more sense for students (Wellington, June 2020).

> I believe that everything can change, facilitate or make students' learning more difficult at all teaching levels, from the early years of elementary school, onwards. From my viewpoint, the Differential Equations Modeling got to adjust this reality to the issue of learning mathematics on a new way (Dione, June 2020).

> This assignment made me seek knowledge, besides that I had acquired in the classroom, it made me see situations that I could not see applied to reality. It was very important and essential for learning the Modeling activities, for learning new things and also for observing how Differential Equations are essential for several fields and applications in daily life (Luísa, June 2020).

The Modeling environment brought along the motivation to learn DE through assignments related to daily life and to the social reality; thus, it has boosted creativity to Mathematics' learning:

> In my opinion, as positive point of Modeling, there was a real gain in Differential Equations learning based on their application and on their interaction with reality. [...] as a future teacher, I have observed that the student learns more as researcher; interaction between teacher and student through research increases, as well as social conviviality (in the case of face to face work); it makes students more motivated and satisfied (Bruno, June 2020).

> As positive point, I think that modeling relates mathematics to the student's reality. The students start solving the problems of their own lives, they get to see mathematics' applicability, and, most of the time, it did not make any sense and after the assignment it does. It also shows that numbers are also associated with social issues such as in the case of the search we made about the Census (Raina, June 2020).

> In my opinion, the main positive point of Mathematical Modeling lies on the fact that, by being carried out, it shows in simple and objective way, to the students, the mathematical concepts in daily life, and it leads to extraordinary learning. As for the negative point of it, I do not think that Mathematical Modeling is the problem, but teachers and students, themselves, because Modeling is laborious and quite tiring to work with, it demands too much work and dedication; therefore, I believe that many professors made the option for adopting assignments that are simpler to develop and evaluate (Bruno, June 2020).

The speech above showed other perspectives about Mathematical Modeling supported by the theoretical reference. One of them highlighted the hard time learning it. This aspect was evidenced by participants' voices, individually and in their groups. Besides, this important particularity is associated with learning and it was also observed by the professor during interactions with the groups; moreover, it was reported in its field journal.

Based on participants' voices and on observations of the assignments, we sought to understand where and how difficulties have happened during DE learning. We boldly state that they emerged from interpretative issues due to the application of mathematical concepts, to gaps in other stages of mathematical learning or to other implicit matters.

Other factor brought us some evidence about learning difficulty based on opinions posted in chats of the Google Meet platform, namely: didactic materials adopted by schools. Other speeches by students also point towards the way to conduct the teaching process, which influences learning; it shows professors' critical-reflexive profile in Mathematics Degree formation – it is important highlighting that it will be essential in future pedagogical practices:

An issue I have seen about Modeling is that most didactics mathematics books, mainly those for higher education, do not provide a more contextualized Math as it happened in our assignment (Dione, June 2020).

> Professor, both assignments about Differential Equations were well elaborated and it was easy for us to find the Law of Malthus. However, we had a hard time finding the Law of Vershulst. [...] Mathematical Modeling should be used by teachers since Elementary School so that the student can have access to this way of learning. It is cool, but we are not used to this kind of assignment and it demands students to work harder, and the professor also has a bit more work. We felt that in this assignment. (Modeling) also showed that Mathematics is present in our lives, but the problem is that, even in higher education, professors fill up the white board [with information] in the classroom, but nor students neither professors are also used with another way of teaching and learning Mathematics (Marcone, June 2020).

> It was possible observing that group G2 recorded a better performance in the resolution of activities inherent to Differential Equations, including graphical analysis and the association of mathematical concepts resulting from Differential and Integral Calculus, such as the horizontal asymptote in the graphic that expresses the Law of Verhulst and the use of the concept of limits concerning the infinite. This group has precisely found the independent and dependent variables, to solve the proposed problem, as well as perfectly described the Law of Malthus. However, it made an algebraic mistake that has stopped it from finding the mathematical law that properly expresses the Law of Vershulst, but it almost got to the model (Professor, June 2020).

However, despite difficulties reported by participants and by the professor, we understood that the Mathematical Modeling assignments have contributed to the learning and re-signification of mathematical contents of DE, and of Differential and Integral Calculus concepts, as well as of other mathematical knowledge required by problems introduced in the assignments.

CONSIDERATIONS AND PERSPECTIVES FOR MATHEMATICS EDUCATION IN HIGHER EDUCATION

Now, we will address some considerations in light of the research on Mathematical Modeling in Differential Equations learning that was herein detailed. Based on such considerations, we will try to provide some perspectives that can become a challenge for on-going investigations about Mathematics Education in Higher Education.

Based on our research, although the meetings took place through remote access, Modeling showed its "identity mark", because it provided an open environment in search of knowledge, given its interdisciplinary profile. Because the assignments were developed through team work, the leaning principles regarding socialization, dialogue and cooperation issues were potentiated. These principles are also necessary for conviviality in society; therefore, for meeting the tolerance premises of knowing how to listen and to behave in groups, based on the very pillars of Education. It is important to have the possibility of building mathematical knowledge in other mathematical-content courses, in different Higher Education courses, mainly by approaching specific problems regarding students' training in their own fields, be it through Modeling assignments, Problem Solving, Digital Technologies or through other methodological trends for teaching.

The research environment provided by Modeling met participants' interests and gave them alternatives, as future teachers, to make "the student autonomous and [to make] the teacher the knowledge mediator". As for the Higher Education context, in particular, and for its teaching practice, it is necessary expressing this mathematical knowledge mediation profile by potentiating its students' investigative autonomy. They must be encouraged to make exploratory research based on the different dimensions setting the reality of what has been investigated. This process may include social, economic, structural policies, among other dimensions that can be approached through problems proposed in the classroom, based on the most varied teaching levels.

Other important aspects of the Modeling environment consisted in bringing Mathematics to the classroom context, as well as in facts of our new generations' nowadays social and cultural context, although some of those factors are not part of the students' daily lives. Putting students in contact with general knowledge was one of the research concerns, since it should be in compliance with the perspective by Burak (2016, p. 34), who highlights the need of students to have "contact with reality". We know that DEs are a fruitful source of application in students' daily lives; however, several other mathematical courses in Higher Education also allow the contact with the real world, at deeper or shallower "theoretical depth", mainly based on the teaching process applied to Differential and Integral Calculus contents, as well as to Analytical Geometry, Linear Algebra and Statistics.

Modeling also substantiated the conjectural conditions - without the expectation of getting ready, finished and single responses. It has shown that the process to seek knowledge is much more important than the final product. We have adapted critical analysis issues related to Mathematical Modeling, whose solutions took place through DE as the way to get to know the size of future populations; it has shown Mathematics as important enough to be taken to the mainstream, as social topic related to the population – it was not addressed in ODE classes – by adopting a critical sight over the perspective by Burak (2002, 2010, 2016). Just as Modeling worked to explore mathematical knowledge and aimed at understanding how future teachers place themselves in criticality issues that were assumingly brought by Mathematics, it seems possible to trigger critical discussions through other teaching methodologies, not just in Mathematics teachers education, but also in the mathematical training of engineers, physicists, economists, educators, among others. After all, this process would be one of the ways to think Mathematics teaching in such a fashion to contribute to the formation of critical and reflexive citizens, mainly in the Higher Education context.

We also observed that the discovery of "theoretical inconsistencies" in answers given by the students, among some other principles of Modeling, must be the object of Mathematics teachers' reflections, mainly in Higher Education. We were inserted in the world of Mathematics as exact science that does not have room for errors. However, we understand that we are not free from difficulties and mistakes in the process to build knowledge, at any schooling level. Thus, Modeling evidenced that we must pay close attention to learning difficulties that, as we have highlighted, can be triggered by different reasons. It is so, because we must have in mind the need of dealing with learning issues in a central and broader way.

Accordingly, we understand that the herein described research brought along important reflections about DE learning, and it has suggested a reasoning about its teaching, which must be rethought based on some of the aspects approached in the study. These aspects substantiated our search for future focuses and topics in investigations on Mathematics Education in Higher Education.

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