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Trends in Mathematics´Education degree: from theory to practice, what are the outstanding reflections?

Tendências em Educação Matemática no estágio na licenciatura: da teoria à prática, que reflexões se destacam?

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ABSTRACT
This article addresses the inclusion of discussions about Trends in Mathematics Education in a Remote Internship course. The article aims to describe and analyze aspects of this inclusion and the consequent actions and reflections about it in interns' practices. The course assignments sought to articulate the theoretical elements of trends that interns mentioned in their reflections about their practice of teaching mathematics remotely. By analyzing synchronous meetings moored on the theoretical references of the Historical-Cultural Theory of Activity, our analysis produced an understanding of two axes that converge on methodologies and teaching moves. We highlight the moves of interns to re-signify the "theoretical" understanding of trends and analyze the possibility of articulating them with their practice. The focus on trends helped interns see themselves as part of a professional category linked to attitudes and values intrinsic to it.

Keywords: Supervised Internship, Trends in Mathematics Education, Remote Teaching.

RESUMO
Este artigo discute a inserção das Tendências em Educação Matemática no âmbito da disciplina de Estágio remoto. O objetivo é descrever e analisar aspectos dessa inserção e consequentes ações e reflexões permeadas por ela nas práticas dos estagiários. Para tanto, as tarefas da disciplina buscaram articular aspectos teóricos de algumas Tendências escolhidas pelos estagiários, promovendo reflexões em termos da prática de ensino remoto de matemática. Por meio da análise dos encontros síncronos, ancorados pelo referencial teórico da Teoria Histórico-Cultural da Atividade, produzimos compreensões em dois eixos que se articulam a um olhar para as metodologias e um olhar para o fazer docente. A partir dessas compreensões, destacamos o movimento de ressignificação dos entendimentos 'teóricos' quanto às Tendências e das possibilidades de articulá-las à prática. O foco nas Tendências favoreceu às e aos licenciandos se entenderem como pertencentes a uma categoria profissional, o que se articula a posturas e valores que lhe são intrínsecas.

Palavras-chave: Estágio, Tendências em Educação Matemática, Ensino Remoto.

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INTRODUCTION

Authors such as Bicudo (2010) and D'Ambrosio (1997, 2004) advocate that Mathematics' Education is a dynamic research and practice field that feed itself. Accordingly, it is seen through its complexity and systematic renewal, which derives from new insertions and orientations that, in their turn, come from practice and research, and that head towards specific aspects. Trends in Mathematics' Education are an expression of such insertions, which point out theoretical perspectives and attitudes that lead to discussions about a given core topic.

Trends in Mathematics' Education, at research scope, allow a sight over some aspects of investigation. They enable opening dialogues to complete each other and to unfold themselves into new demands, due to authors and interlocutors who get involved in these trends. As for classroom practices and actions, and based on our experience as Basic Education Mathematics' professors and as Supervised Internship professors, we have observed that trends not always gain room in formative spaces for critical appreciation.

Thus, the question to be made is: how can the discussion about Trends in Mathematics' Education take place along with teachers' initial or continuous training? This enquiry concerned us, since we are professors accounting for training Mathematics' teachers within a specific context, namely: remote Internship in Mathematics' Degree during the COVID-19 pandemic.

Accordingly, after almost one year of experiences and reflections about students' remote Mathematics' teaching practices, and based on the experience acquired with the first remote Internship, we produced an empirical material which allowed us observing evidences that some of the developed actions were more successful than others; furthermore, we shared our analysis about this experience with the scientific community in Santos and Deodato (2021).

It is important highlighting that during this time we noticed that many students, some of whom are about to graduate, have presented gaps in discussions and experiences on consolidated topics in Mathematics' Education studies. It is imperative informing that, even the ones who spoke up their expectation to keep on studying this field, only engaged in shallow discussions about specific topics in this field.

One of the aforementioned gaps regards interns' understanding about Trends in Mathematics' Education and their pedagogical possibilities in the classroom. We have noticed that interns were interested in getting to know some specific Trends, although some of them did not even use this word to broader topics they have already worked with. Students expressed themselves by introducing more general ideas associated with games in teaching, Geometry in Basic Education, digital inclusion, among other elements.

Thus, we joined forces to organize the second round of remote Supervised Internship focused on Trends in Mathematics' Education. One of the challenges to be faced, and maybe the most significant of all, lied on extrapolating the limits imposed by remote teaching, either in college or in schools that virtually hosted interns, to trigger discussions substantiated by teaching practices and reflections.

A promising pathway headed to the organization of an Internship program that would demand us (professors accounting for training processes), students and higher education professors to propose actions capable of cooperating to Mathematics'

knowledge sharing with students in schools. Thus, we counted on the essential partnership with higher education professors who have hosted and followed-up interns, mainly in in-person internship, and who helped planning the discipline and its organization and conduction.

Accordingly, based on our own past experiences, we aimed at triggering reflections by interns to boost attitudes and world views linked to Mathematics, to its teaching and learning. We organized the discipline to be offered from 2021 onwards, with emphasis on Trends in Mathematics' Education, mainly on those of students' interest; among them, concrete materials in Mathematics' teachings, Mathematics' history, mathematical games, teaching projects, inclusive education, financial and technological mathematics.

The inspiration emerging from this experience allowed us to state that our aim with this article is to analyze insertion aspects at the scope of Internship in Mathematics' Education Degree, as well as in that of Trends in Mathematics' Education and of the consequent actions and reflections supported by them in interns' online practices in order to echo their voices.

Thus, in order to connect to such a goal, and based on assumptions about qualitative-nature investigations, we leaned over the recordings of synchronous classes of the remote Internship in Basic Education (conducted through video conference). Besides, we sought authors from the historical-cultural tradition, who pay close attention to the dialectic discussion between theory and practice, to analyze the produced empirical material.

Nevertheless, we organized the present article into six sections. After the present introduction, which aimed at contextualizing our study, we introduce the elaborations of authors who address Trends in Mathematics' Education and information about the conducted remote Supervised Internship. Next, we introduce the theoretical references substantiating our analysis, as well as the empirical material subjected to it. Subsequently, we make an analysis followed by our final considerations.

TEACHERS' TRENDS AND TRAINING: SOME REFLECTIONS

Mathematics' Education, as study and theorization field, started to consolidate itself in the late 20th century due to discussions and questioning about Mathematics' teaching (D'Ambrosio, 2004). The concern with making the access to mathematical knowledge more feasible to students made professors and researchers seek to renew Mathematics' teaching practices. Discussions linked to Mathematics' Education in Brazil started in the 1950s, but this field only got consolidated in the 1980s, after the Brazilian Society of Mathematics' Education was launched, as well as other study and research centers and groups, which were concerned with Mathematics and with its teaching and learning. In 1993, during the 1st Seminar in Mathematics' Education, it was defined as autonomous knowledge field with interdisciplinary study and research object (Souza et al., 1991). In this case, its interdisciplinary profile is linked to its interface with other fields, such as Psychology, Philosophy, Sociology, among others. Accordingly, Bicudo (2010) points out Mathematics' Education as a broad investigation and action field, so that its interface with several fields highlights the possibility of in-depth studies and actions.

As consolidated field, it allows observing

researchers and scholars' concern with getting critically involved in attitudes and world views associated with Mathematics' teaching actions in studies and research on Mathematics' Education. The aim is to understand how they influence and are influenced by concepts related to this knowledge field and to the part played by this discipline in a 'citizen' formation.

By broadening the discussion involving approaches, methodologies, resources, interactions, epistemologies, history, culture, among other elements, we open possibilities to the direction and perspectives for conducted research, and for those that can result from them, by pointing out some trends at Mathematics' Education scope.

The word 'Trend' concerns will, inclination, intension and disposition. It points towards changes and innovations through a pathway that was initially centered in Mathematics' teaching. Based on its development, and on the whole set of performed studies about it, it was possible broadening the discussions about topics linked to Mathematics, and to its teaching in society, to critical formation, to inclusion, and to other discussions and concerns that extrapolate the classroom environment and teaching/learning relationships within the discipline matrix context. Thus, we can see the opened horizon Mathematics' Education heads to. This horizon gets broader when it comes to its possibility to unfold the dynamics of its own development.

Based on the aforementioned scenario, the process to identify some Mathematics' Education Trends was addressed by Lopes and Borba (1994) back in the 1990s. Among the Trends they highlighted, one finds: Problem Solving (Onunchic, 1999), Computing (Penteado and Borba, 2001) and Modeling (Bassanezi, 2000). With respect to a more philosophical and social interface in this field, Bicudo, Viana and Penteado (2001) paid close attention to the historical view of Mathematics and to the ideology found in discourses associated with mathematical knowledge and with Mathematics in different cultures. Accordingly, some Trends stand out, such as Critical Mathematics' Education (Skovsmose, 2001), Ethnomathematics (D'Ambrosio, 1998; Knijnik, 2006), Philosophy (Bicudo, 2003), among others.

Nowadays, we can observe that reflections about individuals' inclusion in a clearly unequal society and about the acknowledgement of different identities have been triggering discussions about the role of Mathematics in the demystification of stereotypes and in the promotion of equity among individuals. Thus, it is important highlighting the rise of discussions that enable understanding how, where and when Mathematics influences, and is influenced by, social, cultural and political aspects linked to human relationships. Thus, some other Trends stand out, namely: Inclusion (Healy & Fernandes, 2011), Gender Issues (Souza & Fonseca, 2010), Decolonization (Giraldo & Fernandes, 2019), among other.

We think that it is necessary opening a parenthesis to highlight that, despite the fact that it is not our aim in the present article, we have entered the particularities and features of several Trends (be them the herein addressed ones, or others). Therefore, we made the option for making an overview about them in order to contextualize the scenario our empirical study is inserted in.

Hence, this gaze over the investigations and Trends in Mathematics' Education pinpoints its interdisciplinary and dynamic profile, which sets the ways to focus cross-sectioned mathematical knowledge on topics and discussions that are essential to support an inclusive and egalitarian society, where citizens think, argue, make propositions and

change the reality around them. This dynamism is fed by the scientific and education thinking that boosts this process, it works in balance with practice, which is nourished by it. This process enables revisiting the understanding about the learning and teaching process, and the involved individuals.

By adopting the understanding that Trends reflect certain understandings and actions in Mathematics, we corroborate Fiorentini (1995), according to whom, whenever visibility is given to understandings and attitudes in Mathematics, and to its impacts on the way ends and values get related to how this discipline is taught and understood, one can highlight its reflex on actions and relationships in and outside the classroom. This author explains that if Mathematics is considered an exact, finished and done science, the role of professors oftentimes lies on transmitting mathematical concepts and assuming that students are individually capable of leaning based on their own skills – with emphasis on the fact that the "most skilled" ones learn faster than the others. This understanding influences classroom relationships and have impact on the involved individuals; therefore, it is likely assuming that the impacts of a given attitude towards Mathematics' teaching go beyond the classroom and reach several social relationship dimensions.

With respect to the educational scope, identifying and understanding elements related to concepts that support and guide pedagogical actions allows understanding the attitudes and beliefs found in the construction of the methodological and discipline matrix proposition for Mathematics' teaching. Thus, theoretical discussions and productions related to Trends in Mathematics' Education can help developing a broader sight over aspects linked to the developed propositions, actions and practices by pointing towards individual, collective and social impacts resulting from them.

Therefore, based on our own understanding, discussions about Trends in Mathematics' Education must be addressed in the training of students, and initial and future teachers, to provide a wider view over the teaching action and to encourage student teachers' critical view over their practices and actions concerning their own students.

In light of the forgoing, this is an essential view to change the concept bond to "failure" in Mathematics and to the likely potential of the individual, by disregarding the complexity surrounding the learning process. It is also crucial understanding the role of broader actions (and of their outcomes) when it comes to teaching and learning, since it would allow teachers to see themselves as mediators in everybody's knowledge construction, including their own.

It is worth highlighting that, in order to do so, we assessed the Practices and Internship in Mathematics' Degree, where such discussions can take place. Our understanding of it is related to the perception that these disciplines, and their students, have the opportunity to think about and to experience teaching deeds substantiated by a collective space where experiences can be shared and where critical sights can be encouraged.

We aimed at seeking the potential of discussions about Trends in Mathematics' Education in the initial training of Mathematics' teachers. The present article focuses on the interactions allowed by such an insertion. In order to do so, we will dedicate ourselves in addressing the experiences lived at remote Supervised Internship scope; these actions and reflections were crossed by discussions about

some Trends in Mathematics' Education.

REMOTE SUPERVISED INTERNSHIP IN MATHEMATICS

The herein addressed Remote Internship experience was substantiated by our previous experience in offering this discipline within the pandemic context. It was born in remote format and gave us a certain "comfort" in comparison to the first time it was offered. The accumulated experiences made us more confident in the actions taken and allowed us to observe the limitations we would have to face in order to get to promote a space of actions and reflections with the students.

We have approached the first experience and the scenario of uncertainties and re-signification we were inserted in, in Santos and Deodato (2021). The need of social distancing to cope with the COVID-19 outbreak drastically changed the human activities and emphatically changed the teaching institutions. Emergency Remote Teaching, at all levels, brought a lot of insecurity and, simultaneously, made teachers, parents and students rethink their actions and seek ways to cope with the crisis and with the maintenance of both teaching and population health.

As for the university we teach at, Ordinance PROGRAD/UFOP n. 5, from February 04, 2021, provided information for coping with the emergency situation, according to which, classroom classes were initially replaced by class in virtual environment, during the pandemic. There were broad internal discussions aimed at mitigating its impacts on the community, or about all sorts of exclusions that could come up from it.

With respect to disciplines such as Internship and Practices, which concern students' straight contact with collective spaces, these discussions focused on adjusting the remote activities, since the pandemic did not give signs of improvement in its first year. It is worth highlighting that from 2020 - the first year of the pandemic - to the time this article was written, mass vaccination has helped us to get organized to face the pandemic. Overall, the COVID-19 pandemic scenario still shines a red light over us, since it still points towards uncertainties regarding the emergence of new variants and of likely uncontrolled disease outbreaks.

As for the availability of discipline "Practices", at first, it happened on an optional basis and counted on the approval by the involved collegiate in order to be offered in its remote format. A particular aspect of Supervised Internship at graduation level regarded the understandings about lectures, space and time in the classroom, which should be rethought to be adjusted to contexts faced by schools at that moment. Besides, the hard time accessing schools and teachers' work overload - given the need of reformulating the whole planning, in order to fulfil the needs of remote teaching - made discussions about Internships longer when the topic was the feasibility, or not, of having them happening without the physical presence of the involved ones.

Accordingly, the challenged posed to professors and interns was to reinvent the ways to be in the classroom, and it demanded the attention and participation of all, including school supervisors, so that actions would be taken and they would actually involve the interns. Based on the report by Santos and Deodato (2021), this process was marked by stress and by the sense of not reaching basic education students who, at that time, were out of the classroom, mainly in public schools. Therefore, in light of the foregoing, we must point out that the

differences among schools that hosted interns also pinpointed the maintenance of a perverse logic that was identified in Brazilian schools and reported by Libâneo (2012). We refer to the logic that promotes (or reinforces) inequality conditions by maintaining the "perverse dualism" between schools that share the historically accumulated knowledge with quality – these schools are often attended by kids of high-income families – and those that only ensure the minimal elements for students attending them – they are often students coming from the working class.

Interns expressed a sense of "absence" by students, but it was re-signified as we collectively discussed it in the discipline's theoretical classes. Between mistakes and strikes, attempts and frustrations, experiences and learning, after the need of implementing the first remote internship experience, we could observe that the poor participation, or absence, of students was discouraging the interns. By looking at the situation as challenge, students mobilized themselves in an attempt to promote other interactions with their students, and it allowed re-signifying what it would mean to be in the remote classroom. The creation of dynamic workshops aimed at "approximating" students to mathematical knowledge, for example, led to important discussions about the part played by teachers and by digital exclusion.

All these factors made us to think about alternatives for in-depth discussions substantiated by practice when the discipline was offered for the second time. That was the time when we saw the possibility of triggering discussions about Trends in Mathematics' Education as the power boosting the proposed assignments. We saw them as the way to trigger discussions about topics of interns' interest by entering particularities, features and understandings capable of enabling the critical reflection about the classroom and Mathematics.

Accordingly, at first, we identified the Trends of interns' interests and highlighted the aspects of them. Subsequently, the assignments were organized to articulate theoretical discussions about them, based on practices conducted in partner schools.

It was, then, within this scenario that we offered the second remote Supervised Internship, which allowed us to develop the herein presented empirical material. Before sharing the movement towards the produced empirical material, and the dialogues that gave birth to the reflections we sought to highlight in this article, we will address our theoretical references.

INTRODUCTION OF THEORETICAL-METHODOLOGICAL REFERENCES

We have assumed the perspective adopted by Engeström and their collaborators, based on the Cultural-Historical Activity Theory (CHAT) (Engeström, 1987; 2001; Engeström & Sannino, 2010) in order to assess the empirical material we had gathered. It is important pinpointing that, although there is no consensus about it, the study by Vygotskyi and Leontiev – from a broader viewpoint, based on the Marxist dialectical materialism – shows the very fundamentals of CHAT.

CHAT use in Brazil has been common in research on Education (Duarte, 2002), such as in studies on Mathematics' Education (Lopes & Marco, 2015). Appropriations used in this investigation derived from debates we had with members of the research team called "Historical-Cultural Theory of Activity in

Research on Education" (CHATER)"1 (Costa, 2016; Deodato, 2017; Campos, 2018; Souza, 2020; Amancio & Deodato, 2021).

Accordingly, we will now address some theoretical constructs linked to the analysis we have held in this article.

Human activity as analysis unit

We assumed human activity 2 as fundamental analysis unit. We will analyze the phenomenon adopted as analysis object in the current article through this process. We adopted the Vygotskian understanding to build this unit; according to it, the relationship between subject and the object of desire is mediated by artifacts – humans learn from the collective to the individual and, as historical beings, although we are influenced by the features of our species and of the social groups we are bond to, we cannot be determined by them.

The understanding of Activity we have adopted brings along the description by Leontiev – which is often introduced through hunting, for example (Leontiev, 1978) – according to whom, every Activity is collective and it can be observed from three levels. The most general one explains subjects' organization to seek a common object; the second level, the intermediate one, helps understanding the role (and the actions) of subjects in this Activity – each one has its own specific partial goals -; the third level, the most procedural one, explains that individual actions are limited by concrete conditions.

It is also essential informing that we adopted the understating that all Activity is a system (Engeström, 2001) and that, from a pragmatic viewpoint, each element composing it is featured from researchers' perspectives; therefore, it is neither neutral nor definitive. Moreover, this viewpoint is connected to legitimation criteria of a given collective (of a scientific community), thus, in this sense, it is not "individual".

We will use Activity either as design to organize the empirical material, or as analysis tool for the interpretative reading of this material.

Principles substantiating the Activity system: focus on stress situations

CHAT is substantiated by five principles that are described in details in the study by Engeström (2001). Based on our interpretation, these principles refer to the fact that all Activity systems: i) are not isolated, i.e., they set a network of relationships with others; ii) they form a community with multiple voices and viewpoints; iii) they can only be understood from the movement of their history; iv) they are formed by contradictions (source of change and development); v) they have the expanding potential to be transformed by changes.

Among these principles, we highlight the premise that every Activity is a system formed by contradictions that result from stress accumulation, overtime. The point is that stress can historically evolve to contradictions. These contradictions, in their turn, whenever identified and coped with, can be overcome – and it happens in the long-run

[1] Registered in CNPq and certified by this institution. For further information, access <dgp.cnpq.br/dgp/espelhogrupo/7071867555038738>. Accessed on January 01, 2022.

[2] From this point on, every time the word 'activity' is used in our text, it means the addressed concept; it will be used in capital latter and highlighted in italic.

and in rare occasions, according to Engeström and Sannino (2010) – and lead to changes and development in the system as a whole. It concerns the Marxist premise that contradiction is the engine for change.

By following this understanding, we have considered that the tracks of change, which can be found in expanding learning throughout the years, are within the stress situations. Thus, whenever we feature Activity, we focus on identifying such stress situations.

Triangular model as analysis tool

It is imperative pointing out that one of the greatest challenges faced by researchers is related to the demand for operationalizing concepts in such a fashion that they become "powerful" analysis tools to read the reality. We adopted the triangular model by Engeström (2001) to face this challenge.

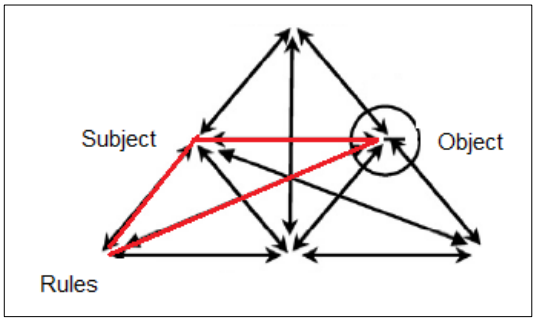
Although we are aware of the critiques towards this model – mainly because it is assumed to produce a "motionless" description of Activity -, we will use it, since, in our opinion, it somehow helps organizing the empirical material. Furthermore, it also allows finding the focus of the analysis, with some degree of accuracy. We will adopt a careful description to mitigate the assumed loss of the motion issue (of featuring as 'photograph' what is actually a 'movie').

This model was introduced, described and used in several studies (Souto, 2013; David & Tomaz, 2015); it is based on Vygotsky and Leontiev's ideas, but it has broadened them, since they added more details to the social basis of Activity, namely: rules, community and division of labor.

Focus on the mediation set through component 'Rules'

We will herein zoom the triad 'subject', 'rules' and 'object', which is highlighted in the triangular model, as it can be seen in Figure 01, below. We will focus our sight over component 'rules', mainly over the insertion of a new rule (Trends in Mathematics' Education) that mediates subjects' (students') relationship with the object of Activity (teachers' training).

Figure 1 – Adapted Triangular Model, with highlights



Source: Adapted from Engeström (2021, p.135)

It is also important highlighting that, based on this model, subjects are participants whose viewpoint is adopted as analysis perspective; the object is the collective reason connecting each subject to Activity; rules refer to "... regulations, standards, conventions and patterns, either explicit or implicit, that interfere in actions in the activity system" (Engeström & Sannino, 2010, p.6).

Briefly, we aimed at using this focus to shine light on the mediation of a new rule that was appropriated by subjects and that ended up getting them closer to the object of Activity, as such an approximation was happening.

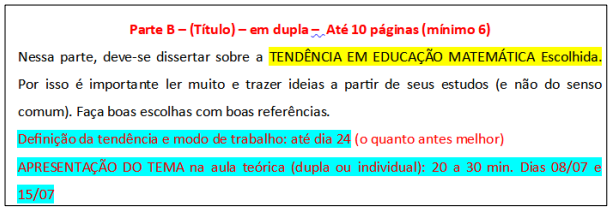
INTRODUCTING THE EMPIRICAL MATERIAL

The herein emphasized empirical material comprises the excerpts from transcriptions produced from the provided lectures, in synchronous format, at the scope of the discipline Supervised Internship in Basic Education.

Furthermore, it is crucial informing that we have revised the classes' recordings and selected the excerpts that would help us reaching the aim of our research – describing and analyzing aspects of insertion at the scope of the Internship discipline, of discussions about Trends in Mathematics' Education and, consequently, of actions and reflections substantiated by them in internship practices, by opening room for the students' voices. It is also important pointing out that Internship classes were provided once a week and they lasted 2 hours.

The first three meetings were organized to most often explain discipline planning, as well as to share information about how demands related to Internship documentation, in remote format, would be. It is imperative highlighting that we have observed that most of these students had already attended the Supervised Internship discipline in the semester before. Therefore, they were, somehow, familiar with remote classes. Another necessary highlight lies on the fact that we introduced students' main critiques to the previous Internship, right in the first class. Based on these critiques, we would propose a new structure to the Internship process. The two main identified claims were related to i) the demand for a less technical report format that could be capable of better describing the remote scenario and ii) the possibility of adopting teaching methodologies that could better help describing – and developing action in – the remote classroom. A third highlight is the fact that we have addressed the structure of the Internship report in details; at that time, we emphasized that all students would have to choose a Trend in Mathematics' Education to guide the construction of their report. We have explained our expectations concerning the report, based on a structure – organized within four blocks – that should be previously prepared and made available in the virtual platform of this discipline. Just as illustration, we introduce a part of this structure, below (Figure 02).

Figure 2 – Explanation about the Trend in the Report



Source: Researchers' inventory (2021)

When we introduced the expectation towards the report, students also had several doubts about it, and three have stood out among them: i) Are we free to choose a Trend? ii) Must the teaching practice be related to this Trend? iii) What can we do in case the school does not agree with our choice? We set collective agreements based on these doubts in order to establish the strict boundaries for the understating of what a Trend would be. Accordingly, we aimed at respecting interns' freedom to choose the one they were interested in. Moreover, we made them free to use the chosen Trend in the teaching practice, be it for personal will or for school availability.

After these initial classes – which were featured by a more managerial, or sometimes technical, profile -, we started addressing some Trends to provide students with

references to substantiate their choices. Thus, we have elaborated productions about Mathematical Modeling (Carraher, Carraher, Schliemann, 1982; Knijnik, 2002; Lave, 2002), Games and Manipulative Materials (Fiorentini & Miorim, 1990; Grando, 2000; Deodato & David, 2015), Gamefication (Zichermann & Cunningham, 2011), Problem Solving (Polya, 2004; Allevato, 2005) and Mathematical investigation (Ponte, Brocado & Oliveira, 2003).

It is worth pinpointing that students were informed that, besides choosing a Trend and assessing it, they would have to produce a video introducing it to the classmates. They were also informed that the Trends we addressed were only examples and that, therefore, they would have the possibility to propose new tasks and analysis for the discipline.

Other two classes were used to work with the workshops we had developed based on experience, to discuss the aspects that we had as essential to mathematics' teaching, and that, on our opinion, could exemplify the likelihood of working with mathematical contents in the remote format. The following topics were addressed in these workshops: i) areas and perimeters based on investigations on virtual Geoplane; ii) Multiples and divisors through games in virtual environment. It is important highlighting that they were asked to think about what trends were related to the workshops in these classes.

Students in the two following classes introduced the trends they had chosen; namely: financial education, ethnomathematics, the use of games, mathematical modeling, the use of multiplicative resources in geometry teaching, mathematical investigation and inclusion. It is important pointing out that there was a previous agreement that each student, besides preparing a video, would be in charge of assessing the video about the Trend chosen by other classmate; in other words, each student was accountable for weighing about, at least, two trends: their own and that of a classmate. Furthermore, they had access to the synchronous debate about all chosen trends.

Then, three classes were organized for students to get to know professors' experiences with Elementary School education. In one of these classes, they talked with a professor about the intercultural training of indigenous teachers. In another class, they got to know the experience of a Mathematics teacher who works in Elementary school and in High school, in the municipal and private networks of Belo Horizonte City, and in the public federal network in Minas Gerais and Rio de Janeiro states. Finally, they could talk with teachers who work with Mathematics in Elementary School, in the public state network in Ouro Preto City.

Two of the last classes were organized for students to introduce their internship reports and to the individual understanding of evaluation and feedback. The other two classes were the space for students to address how was the experience with the remote Internship process in the school they were acting in. This report took place in the first class, based on the production of a letter – to report to a freshman in the Mathematics Course the feeling about working in remote classroom. The reports got a shape in the second class, based on the debate guided by enquiries about the role of Trends in their practical experiences. Right after that, we selected two excerpts from this second class, which were produced when the professor asked whether the students had used the Trend in their teaching practice or in their teaching-practice planning (excerpt 01), and how they evaluated the Trend's impact on

their Internship (excerpt 02).

Chart 1: Excerpt 01 from class 11, on August 05th, 2021.

Student D: I don't know the trend…

Student A: Did you use the book? You mentioned that the professors use it a lot

Student D: Thanks (student A) for thecollaboration… I know the book, but… I added some elements to my class in order not to use the book all the time too… But I sort of got a script that was in the book the professor has indicated.

(…)

Student D: The trend that I wrote about was Ethnomathematics and I didn't use… Yet, I believe that it was because of the issue I had mentioned at the beginning… because of personal issues I didn't have too much time to organize it, but I believe that there are many cool points that can be used as Trend.

(…)

Student R: I am the partner of (student) V... V (Student) is in the other class, right? We attended, on the 09th, we attended a workshop about angles and triangles… we presented the content and at the end we were going to do… we did, right? We made a mathematical investigation with the paper folding workshop to find that the internal sum of angles in the triangle is 180°… but… What happened… (…) Well… We also used video resources, slides and GeoGebra… But… We did not get to reach the end, which is the most important part…With time… Then, we performed it too fast, you know? Then we decided to do it as second time...We… Professor R said: "use our time on Wednesday"... when it is her meeting…Then, we did it again … then, we reduced the activity in the plan we had prepared, you know? I got more time to apply this investigation again (…)

Professor: You mentioned the trend… That in your case de Trend was used, right?

Student R: Yes… It was [hesitating voice]… That was it, but it was not quite as …

Professor: That is what we want to know … tell us…

Student D: I was curious…

Student R: Oh, because I mean… At the first time it was too rush and I… Gee, I am desperate with time, I started talking to fast … "Oh, just do like that" [simulating the folding with his hands]… I didn't say they should bend in the median… I just did "that's the way it is people" [once again simulating the folding with his hands]… Then the kids got lost… If I had a little bit more time, if I was more relaxed too… Just as in a normal ordinary day … It would be very good... But, then, in the second time [class] it went on easier… The kids were turning the camera on, asking all the time: "is it like that?"… Then it worked much better than at the first time… At the first time, I think it... You know? And [long pause]... I think we need... I don't know... I think we need more... Time... To apply it... Or one whole class to apply it... Then have a content class before it... I don't know, that is what I thought.

(…)

Student L: … Just to contextualize, I worked with them in the virtual Geoplane… The one the professor introduced to us in the classroom… Then I adjusted the Geoplane... One...One... It is... A kind of investigation, right? About the Pythagorean Theorem (…)

Professor: And in your case… Then… Did you use the Trend you are assessing, L.?

Student L: Well [instable voice]… an adaptation, actually, right? Because my Trend is manipulative resource, right? And as we are in the online context, I had to use the virtual Geoplane that... That I see as a manipulative thing, as well, right? You can handle things there…So, this way, we can't… I couldn't use the physical Geoplane, right? But I do think that I have used my Trend.

Source: Researchers' inventory (2021)

Chart 2: Excerpt 02 from class 11, on August 05th, 2021.

Professor: Getting to know these Trends

that you did… So… Knowing that there is Ethomatematics…Knowing that there is Financial mathematics… Knowing that there is Investigation... Yes... Knowing that there is Gamefication... Has it helped you somehow? And you can be free to say: "no, it did not have any applicability"… Or, did it help? What do you think? Knowing about the existence of Trends, you know, did it help the practice, or not?

Student R: I think it helped, you know? Because as we are still in the initial training process, it is another possibility, right? Well…We can use it, that it can use one more… I think that… Wow … I guess… I read it… I'm having contact with this word (Investigation) since when I first got to college…Since I got to Mathematics' Education…But I just started reading about it now…You know… I loved the way that... How everything is thought of... How everything makes sense, right? I think that it is a quite powerful weapon... It helped me a lot...

Professor: So, just for me to understand… The motivation to conduct the practice was the reason for you to lean over the Trend…Is that what you are saying: because you already knew the word (Investigation)…

Student R: Yes, I read and I said, boy! This is very good! I mean… The guys talking, the good guys... Just like Ponte (João Pedro)... I said: "Gee"... And applying it was also very cool... I also feel like... If I had a class of my own... I have it already, right? But...When I will be, like, not a student, but a real teacher… I was like, just thinking: "I can do it, I can use it in here, I can use it there"… I thought it was quite "awesome"...

Professor: I'm teasing R because... We were chatting... I already read the plan... I will play the devil's attorney, ok? It goes like that... You said you think the Trend nailed and so... But before you had told us that the Trend was short in time for you… That is what I'm trying to understand... If the Trend helped or if the Trend didn't... what is it about?

Student R: I see! I thought you were talking about life, not about the class… But, yes, it helped… It just did not work quite as I thought it would, you know?

Professor: So, I leave it open... Maybe… Without judging if it is good or bad… But we are trying a new idea… a Trend… For many people it was the first time…So… Dealing… What is written in the text is quite different from what we experience, isn't it so? And this is what you said (Student R)… We are recording, I can't even use that word… This guy here is (badass)… Kind of Ponte, as you wrote there and so on… [students laughing]… It is all there, step-by-step…But, at the time you will apply it, is it written?

Student: No... It is not written...That's the point, right? [the student laughed]… But I guess that ... If I apply it... Again... I think that... Well... But, actually I applied it again, right? [referring to the fact that he had applied it twice]… [laughter]… I guess it would be better, as it was...

Professor: I think that what you are saying makes total sense… I'm stating it just because, maybe, to think about it… like that… Sometimes, we also need to get used to the reaction... It is almost that thing… The student needs to get used to this other way of attending classes… Then, when it gets used to it... Because, at the first time, when you do something different, you take longer to understand how it works, isn't it so? Then it takes longer…

Student R: And the online thing, right? It is more complicated …

Professor: That is it... This is something I think not even Pontes had done before… I mean, online Investigation... At Elementary school... I think this is something you are innovating at, you see? [laughter]

Student R: Yes, it is totally different from what I was used to…

Professor: So, everyone, what else?

Student L: That is what happened to me, right? Because I chose a Trend… And, then, I started reading about the Trend, I stopped and I thought: "My Gosh, but what is this Trend in the remote context?"… What is it to use the multiplicative resource in Geometry teaching based on the context we are in now, you know?

I thought that I couldn't for example… Yes... Demand students to have, for example, a straw at home for us to do something … To have something, you know? Then, that was what I started questioning myself about the Trend, as well, you know? Then I read somethings… Then, when I decided to use the Geoplane I said: "but, no, the Geoplane, no matter that it is a technological resource, it is also manipulative… You know? Then… It was herd because… Despite the fact that you have to… Assess the Trend… We also had to adjust it to this context we are in now… So, we had assignments to do...

(…)

Student A: I'm curious to get to know how it is like… I mean… How we will deal with the coming internship, like… Well, if we will keep this Trend thing… We can go on with it… It occurred to me now... We can go on... With the same Trend or do we have to change? Well… And an observation about my trend is that it is basically the same thing, right? Just that… Based on what is written there, it is quite... It is quite nice... In the official website we have accessed... Even in the ENEF3 video is all very beautiful... You don't use even one third of what is written there… because in practical terms, many other things pass on our mind… so, It is complicated…

Professor: In your case, do you suppose that... Because you still did not see! [the student had her plan ready, but she had not applied it yet]

Student A: No, what I'm saying is that… Because at the time to develop, like, the workshop… there is a lot of written things in our class plan that… That is more detailed... Think about a more synthetic thing, then on a broader thing, I don't know… But, at the time to develop the workshop... For you to leave the basics, well, it is too difficult to explain… Because everything has a way that has to be explained... Well, let's talk about Savings… Let's talk about bank account… Then you have to talk about Selic rates… Then you have to explain to them what Selic is… And there is some curiosity about it… We focused on its benefits, for example, check books… then the professor gave us a tease: "the kids have a great chance of not knowing what a check is"… then you have to explain what it is… So, I mean, in practical terms, it is much more…There are too many details, you know?

(…)

Professor: But, it is so, because I wanted to understand the difference about working with Trend… To think the teaching plan with and without the Trend… To think about the experience with and without the Trend... What did you think about this move? Do you think that it has affected your way of thinking, or not? And what about the practice?

Student A: am I supposed to be honest?

Professor: Sure!

Student A: I though it made the report much harder… I mean… Because as I was here, you know, in the last semester… As we did not have much of this Trend thing, and so on... we only had to focus… I go like that… all or nothing… Because... As I attended the last semester and there was no Trend, I followed the basic, as it was always, before… Keeping what B [supervisor professor] did in class... For example, I followed the book, I followed the handbook and I did it… As I did not apply it, yet, right? I'm not so sure to talk about it... But... now, in this internship, I did not go by the book… It is another thing... It is another world because of the Trend... But I think it was much harder...

Professor: Does anyone disagree with A, anyone who agrees with her? Who has the before and after experience?

Student L: I was here thinking about... Comparing my two class practice experiences… from the last semester to this one and I have the feeling that I was already working with this Trend in the last semester, because my teaching experience… I don't know if you will remember, professor, it was about area, right? Then I went

[3] https://www.vidaedinheiro.gov.br/ < Accessed on March 16th, 2022>.

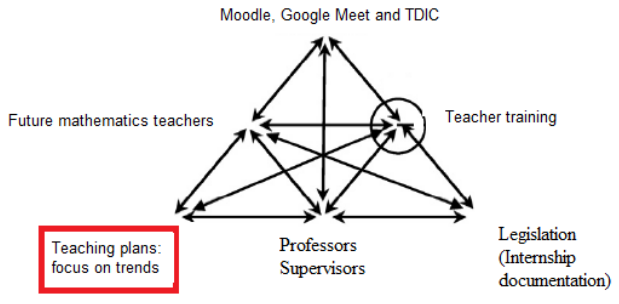
to the Geoplane to show the "little formulas" about in there with the kids... Then, well, I have the feeling that there is some resemblance between the two and by comparing it to the other classes that I attend, that it is just solve the Tutorial Study Plan... I feel like these two classes I lectured using different things... I had the feeling that students liked it more, you know: Thus, for example, after the classes, they sent me a message in the group saying that the class was cool and so on... And, every time it is a TSP solution class, they never do it, you know? So, I think that they enjoyed this class type a lot.

Source: Researchers' inventory (2021)

ANALYSIS OF THE EMPIRICAL MATERIAL

By leaning over the previously presented scenario, we observed some recurrences we disagree with. It is imperative highlighting that we used the CHAT theoretical-methodological reference and counted on the support from the triangular model by Engeström and Sannino (2010) (Figure 3). Thus, we built an analysis unit within the group of the previously described synchronous classes.

Figure 3 – Internship Activity



Source: Adapted from Engeström (2021, p.135)

We identified the Internship Activity in this group of classes, whose object was teachers' training. The subjects were Mathematics Students and the community encompassed Internship professors and supervisors. The measurement artifacts, due to the remote context, were the Moodle environment (articles, tasks, among others), communication service Google Meet (synchronous meetings), applications and software used on the horizon of Digital and Communication Technologies (GeoGebra, Geoplano Virtual, WhatsApp, among others). Division of labor was established at university scope, based on the internal regulation of Supervised Internship, which is more globally guided by the federal legislation that regulates this regime. Component rules, in their turn, stood out in agreements set in the teaching plans of discipline Supervised Internship in Basic Education.

Accordingly, based on this scenario, we will focus on the relationship between subjects and the object of Activity; this relationship is mediated by component rules, which is understood, as already mentioned, based on elaborations by Engeström and Sannino (2010). This focus is justified by the fact that it was inserted in a new rule – that we referred to as rule "Trends" in the teaching plan of discipline Internship, namely: students should organize observations and teaching practices based on reading a Trend in Mathematics' Education, which could be freely chosen. It is worth highlighting that we organized two analysis axes, which we have addressed.

Theory and practice: sight over the methodologies

We point out that the insertion of a new rule in the Internship Activity to stress the relationship between subjects and the object of teaching practice showed that students

have associated the chosen Trend with Mathematics' teaching methodologies. This association stopped some of them from thinking about the material conditions to make the use of this Trend feasible (or not) in the practice they have adopted in the schools.

The actions by Student L have shown that the choice for the Trend, initially because of the new rule (because I chose a Trend…And, then, afterwards, I started to read about the Trend), made her think about the possibility or organizing her teaching practice, given the concrete condition she was inserted in (what is it to use the manipulative resource in Geometry teaching by thinking about today's context you know?). It is possible stating that she not just got to know new artifacts in this process, but also made use of them (Smolka, 2000), in other words, she took them to herself (just to contextualize, I worked with them in the virtual Geoplane… the one the professor introduced to us in class…then I adjusted the Geoplane). Apparently, this appropriation was influenced by the contrast between planning and the use of the artifact, given the concrete conditions of reality (and then, when I decided to use the Geoplane I said: "but, no, the Geoplane… it is also manipulative").

This movement of thinking about the Trend as methodology was subject of speech in the narratives of other students and it showed that it derived from the readings, from the time studying the subject, from reviews of past experiences - it was in contrast to the remote reality. Thus, although with similar features, it was screened in different ways by the students. While it seemed 'challenging' for student D2 (it is a great challenge to adjust the Trend in the virtual…I think that it is very… very… challenging, indeed), it was 'hard' for Student K (the Trend issue was harder… there is a whole preparation) and for Student A (it is another world just because of the Trend… but I thought it was much harder).

It is important pointing out that this "hard" and "challenging" process resulted in elaborations that seem to be evidence that the stress found in this process helped changing the students. Student L, after the readings and reflections, acknowledged that the chosen Trend was part of her practice (by comparing my two teaching practices…From the past and this one and I have the feeling that I was already working with this Trend in the last semester) and she also accumulated a repertoire to think about alternatives to the ways to teach Mathematics. These new ways are more recurrent than those she had found in remote classrooms (by comparing the other classes that I attend it is simply solving PET exercises… I feel like these two classes I have used different things… I have the feeling that students liked it a lot, you know?). Furthermore, it has shown that her statement was based on evidences substantiated by observations of students' evaluations (Thus, for example, after the teaching practices, they sent me messages in the group saying that the class was cool and so on…And when it is a PET class, they never do it, you know?).

Student A, in her turn, not just stated that the Trend has affected her online practice, but also brought up examples to illustrate her statement, and it contrasted her experiences with and without the Trend. According to her, without the Trend, she made the option to 'follow the basics' (keeping what professor B [Supervisor] did in class…For example, then, I followed the book, I followed the handbook, that is what I did), and after the Trend, she felt safe to think about situations that could extrapolate the didactic book (but now, in this Internship, I fully escaped the book… It is another thing…It is another world just because of the Trend).

Student R, in his turn, showed his interest

in the chosen Trend, which was moored on his personal background in the university, he also showed to have leaned over the topic to understand the chosen Trend (I'm having contact with this word (Investigation) since the time I first got to the university…Since the time when I got to Mathematics' Education… But I really started reading now). By doing so, he seemed to have given a personal meaning to what he has read (I read it and I said, boy! This is very good! Like… The guys talk, the good guys…Just like Ponte [João Pedro da]…I said: "Gee!"). Besides, he seemed to have gotten to know and to valorize the epistemology of a given way of understating the Mathematical Investigation (I thought that it was very cool, you know? I think that it is a very powerful weapon, really). Yet, he showed a critical and conscious view, that all these 'qualities' in the Trend were limited by the remote context (and online too, right? Then, it is more complicated…).

Therefore, on the one hand, by using the understanding that stress situations are the seeds of changes – changes that, whenever accumulated and coped with, favor the development of expanding learning process -, but that, on the other hand, by adopting the actions identified in students' online practice, we can think that Trends' insertion, as a new rule (Engeström and Sannino, 2010) of the Internship Activity, makes students compare the readings to the objective conditions of reality - it emerged as powerful tool to favor the training of these future teachers.

Theory and practice: sight over the teaching occupation

As highlighted in the previous section, the insertion of rule 'Trend' has stressed the relationship between subjects and the objects of teaching practice. This stress resulted in learning processes about the occupation of teaching when it is faced by students.

A first learning refers to understanding that the occupation of teaching requires attention to the difficulties emerging from the "practice" – with respect to the observed case, they were imposed by the reality during the pandemic; in our opinion, they revealed aspects of the broader context where part of the students in public schools are inserted in. Besides, it also refers to the understanding that these difficulties can affect teachers' planning and their performance in the classroom – or, in times of remote education - or on the screen. This statement is illustrated in the speech of two interns.

One of them was Student L; she acknowledged that it is necessary weighing about difficulties that can dialogue with the broader context the students are inserted in, when teachers plan their activities (I think that I can't, for example… Well…Demand students to have, for example, a straw at home for us to make something…This type of thing, you know?). She also showed that the classroom - therefore, a more practical dimension - made her rethink about the understandings made possible by the performed readings, which brought along a more theoretical dimension (Then, it was at this point that I started to question myself about the Trends, you know?); in other words, it has shown that elements of practice made her rethink the theory. When it comes to CHAT, the student evidenced to have faced the stress caused by the rule "Trend", as well as showed that this rule ended up mediating her relationship with the object of the Internship Activity, namely: teachers' training.

Student D, in her turn, showed that the rule 'Trend' only made sense when it was experienced in practice (then, I mean, at the time… The students had a hard time accessing it…They did not get to login [in the virtual

game]), in other words, she showed that the rule 'Trend' – at first, it was apparently emptied from personal sense, it was just a task proposed by Internship professors -, started helping her understanding the observed reality (Actually, it made me see that inside the classroom, there is all of it that if I had not assessed the trend… I mean… It would not be so clear in my mind that it is happening in the classroom). Therefore, it has helped her in her training as teacher, i.e., it has collaborated to her relationship with the object of the Internship Activity.

We must add that, with respect to the speeches of students' L and D, they have pointed out the difficulties that can be shining light on aspects specific of the remote context – such as the impossibility of families to provide materials typical of schools in their homes or even the instability of fixed or mobile networks in the internet – in our experience; we have observed that they are no longer general, i.e., they mainly affect students in the most vulnerable social context, mainly those who attend municipal and state public schools. When we aim at the literature, our experience does not emerge as an exception to the rule, but as a recurrent example. Thus, Kohan (2020) states:

> Let's observe some of the main aspects related to this school that we did not observe so clearly and that this time of pandemic has clearly shown (the relationship is not exhausting): the radical difference between public and private schools and, overall, between public education and private education; the large amount of things that are done at school, that do not regard only teaching and learning, but the school's social dimension in a country like Brazil (…) the significant inequalities of the Brazilian society, a large part of the population does not have the minimal connectivity and equipment conditions (…) (Kohan, 2020, p.5).

This statement, which substantiates the experiences we live, is not at the service of "demonizing" the public school but, actually, of acknowledging its core importance to the construction of a State aimed at facing its inequalities. As stated by Kohan (2020):

> (...) an attentive sight over the effects of the pandemic show, opposite to the voices that advocate for the end of the school want to conclude, the extraordinary and unreplaceable value of school as historical and social institution, and as way of suspension and profanation (MASSCHELEIN; SIMONS, 2014), allows those who are inserted in it to put the world on the table in order to put it into enquiry, to understand it, to problematize it and, maybe, to imagine and experience other worlds (Kohan, 2020, p. 6).

Another identified learning regards the sense of belonging to students' professional category. Student R, when he was asked about his motivation to study the Trend 'mathematical investigation', seemed open to the possibility of using it in the classroom, as well as evidenced his sense of belonging to the teachers' category (I also feel like… If I had a class of my own… I have it, right? But… When I will be like that, not as a student, but as a real teacher). It is important highlighting that this feeling concerns a construction process, since the student seems to look for using the 'Trend' in his life, in the classrooms, as teacher (OK! I thought that you were talking about life, not about class… But it helped me so). Based on CHAT, this feeling evidenced by R points out that the rule 'Trend', assumingly due to the flexible profile it proposes, seems to help some subjects to walk towards the object of the Internship Activity.

The third identified learning regards the understanding that the occupation of

teaching is benefited by the dialectic understanding of the theory/practice relationship. Student R, for example, seems to have realized about the benefits of this relationship, when he faced the teaching practice challenges in loco. He mentions that he can see the positive outcomes from Trend studies in his experience with remote teaching, although the implementation of it was not exactly as he had though (But it has actually happened… It just did not happen the way I was thinking it would be, you know?). Besides, he acknowledges that the complexity of the practice is not fully described in the theory (No… No, it is not written! That is the point, right? [Student R laughs]). Accordingly, he seems to acknowledge that the theory helps improving the practice. He also shows himself interested in carrying out practices inspired in the Trends (If I applied it…I think that… Well…But I have applied it, right? [referring to the fact that he had applied it twice] …[laughter]…I think that it would be better, as it was).

Student A also seems to observe that the relationship between theory and practice have positive outcomes in teachers' occupations. Different from Student R, she observes it based on the reflection developed during the teaching practice planning. The student seems to acknowledge that the theory helps predicting the practical issues (there are a lot of things written there in our teaching plan that…That is more detailed). Thus, she not just acknowledges that the practice stresses the theory (Because everything has a way that you must explain…Oh, let's talk about Savings… Oh, let's talk about Bank Account… Then you reach the Selic rate…Then you have to explain them the Selic…Then there is the curiosity), but also that the theory cannot describe the practice in its whole (What is written there is very nice… It is all very beautiful; in the official website that we have used…Even ENEF video is all very beautiful… You do not use one third of what is written there… because in practical terms there are many other things that cross your mind).

Students' elaboration, be it based on the teaching practice bias or on the teaching practice planning bias, suggests that the rule 'Trend' - as it is getting appropriated and, consequently, understood as meaning by the subjects - makes them closer to the object of the Internship Activity.

Thus, based on the aforementioned traces, we have considered the possibility of stating that the rule 'Trend' has stressed the relationship between subjects and object and, when the stress resulting from it was faced, it enabled students of the Internship Activity to have access to dimensions regarding the occupation of teachers, such as: actions thought based on the students, on belonging to the teachers' category, on understanding the dialectic between theory and practice. We also considered possible advocating that stress coping seemed to have being favored by the flexible profile of rule 'Trend' – since each student could choose the Trend they would lean over and assess what would be the possibilities and limits to articulate it to their practices with their own students.

CONSIDERATIONS

We herein aimed at describing and analyzing the aspects of insertion at the scope of discipline Remote Internship, of discussions about Trends in Mathematics' Education and, consequently, of actions and reflections surrounded by them in interns' practices, by opening room for students' voices. In order to give answers for such a goal, we leaned over the experience of triggering discussions about Trends in this discipline. Thus, it was necessary mobilizing

actions to make students reason about the ideas they found in the texts and about the practice they have performed in the schools they remotely acted in.

In our opinion, the possibility of students choosing the Trend they were comfort with and thinking about it during different tasks was evidence that there is room in Internship disciplines to such an insertion. Thus, by focusing on their school experiences and by reasoning about the possibilities and challenges of the online practice, due to the chosen Trend, they got to observe the different shades of attitudes and ways of being along with their own students and Mathematics.

We have used the assumption that trends in Mathematics' Education can inspire theoretical-methodological actions and reflections by creating pathways for Mathematics' teaching in Basic Education. As we see it, the challenge lied on thinking them in remote teaching, without students' physical presence. The search for giving online classes (workshops) substantiated by the chosen Trend pointed towards the fact that other perspectives and guidelines emerged from the remote education scenario: the investigation would require some agreements with the students: the game would be played online, and there would be the need of broadening the manipulative material, among others.

We have observed that, either remotely or in person, there are obstacles to be faced, but the recorded results from the discussions substantiated by practice were promising for those subjected to the action. More specifically, we used CHAT (Engeström & Sannino, 2010), and, by leaning over the Internship Activity, we observed that Trends, by helping students to compare the readings to the objective conditions of reality, emerged as powerful tool to favor the training of these professionals. Accordingly, it was possible highlighting interns' movement to re-signify a workshop, or even the possibility of thinking out of the didactic book.

We also realized that, after analyzing the rules (Engeström & Sannino, 2010) in an Activity system, Trends showed the potential to help students accessing some dimensions of teachers' occupations. Some of them, for example, seemed to acknowledge themselves as part of a professional category and to understand that an unsuccessful experience in this profession does not mean that this pedagogical action cannot be successful in another context, with other subjects.

When it comes to an indicative synthesis of aspects related to teachers' initial formation, we highlight that students started to propose and to carry out remote teaching practices featured by the addition of methodological strategies other than those practiced in the first remote Internship and supported by attitudes and views concerning Mathematics that are linked to topics chosen by them. The discussion about a giving Trend allowed reestablishing understandings, highlighting similarities and differences between them, thinking about the teaching practice and reflecting about the needs and obstacles bond to them, as well as about the possibilities to provide more significant learning process to Basic Education students.

Despite the fact that we do not have conclusive data, we highlight the intention to point out aspects to be investigated in-depth and that, from our viewpoint, as Internship professors. We have observed that Trends' insertion made some students build an Internship report based on deeper analysis. We noticed that this instrument stopped being acknowledged just as bureaucratic or evaluation task of the herein addressed discipline, and reached the position of a critical-analytical journal of the practice itself.

Reading reports aimed at articulating the theory in the texts about Trends to practice pointed towards an academic growth among students.

Thus, by collectively and collaboratively focusing on the Trends, and by understanding these concepts surrounding them, and how they could be implemented by the students in the schools they were acting in, it was possible to go on with discussions about valorizing the teaching spaces and about the importance of training teachers, and about the role of school in the formation of a fairer, more inclusive and egalitarian society.

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