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Reflections on the development of mathematics teaching in France and Brazil: brief history, characteristics and contributions

Reflexões sobre o desenvolvimento da didática da matemática na França e no Brasil: breve histórico, características e contribuições

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ABSTRACT
In this text, we reflect on the emergence and development of the didactics of mathematics (DDM) in France and Brazil. We highlight features of this trend and illustrate some of its contributions to mathematics education and educational sciences. DDM emerges as a scientific field in the 1970s. In its origins, the theory of conceptual fields (TCC) and the theory of didactic situations (TSD) have a central place. Still, DDM currently aggregates a great diversity of theories and methodologies in which the questioning of objects of knowledge and the intention to change relationships with knowledge are relevant elements. Scientific interactions with Brazil have intensified since the 1990s, and since 2015, Brazilian researchers in DDM have been meeting at Working Group 14 (GT 14) of the Brazilian Society of Mathematical Education. In addition to theoretical evolutions of the concept of environment, it discusses, from the perspective of the anthropological theory of didactics, the change in paradigms and the relationship between the teaching problem and the research problem.

Keywords: theory of conceptual fields, theory of didactic situations, anthropological theory of the didactic

RESUMO
Neste texto refletimos sobre o surgimento e o desenvolvimento da didática da matemática (DDM) na França e no Brasil; destacamos características dessa tendência e ilustramos algumas de suas contribuições para a educação matemática e as ciências da educação. A DDM surge como campo científico, nos anos 1970. Em sua origem, têm lugar central a teoria dos campos conceituais (TCC) e a teoria das situações didáticas (TSD), mas a DDM agrega atualmente uma grande diversidade de teorias e metodologias nas quais a problematização dos objetos de saber e a intenção de modificação das relações com os saberes são elementos relevantes. As interações científicas com o Brasil intensificaram-se a partir dos anos 1990 e desde 2015, pesquisadores brasileiros em DDM, reúnem-se no GT 14 da Sociedade Brasileira de Educação Matemática. Além de evoluções teóricas do conceito de meio, discutem-se, sob a ótica da teoria antropológica do didático, a mudança de paradigmas e as relações entre problema docente e problema de investigação.

Palavras-chave: teoria dos campos conceituais, teoria das situações didáticas, teoria antropológica do didático.

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INTRODUCTION

We begin this text by explaining some of our positions, forged in our professional and personal trajectories, in which didactics of mathematics (henceforth DDM) plays a central role.

Among the reasons why we chose to be professors and researchers in mathematics education is the understanding of education as an essential factor for individuals to have autonomy and fully exercise their citizenship. We understand that there is no single, universal mathematics, devoid of influences from the world and the circumstances in which social practices are experienced. Mathematics is plural, and all the different manifestations of mathematical activities and knowledge must be respected and can be studied. We defend that school education must take the role to create favorable conditions for a plural and diverse human coexistence, for the development of a critical sense and for the overcoming of colonized and colonizing cultures. A narrow view of mathematics does not serve this project. On the other hand, we think that ignorance of hegemonic mathematics works as an important factor of social exclusion. So, among the focuses of interest that occupy us, there is one that concerns the creation of favorable conditions for the learning of hegemonic mathematics by all, including in the scope of school education.

We consider DDM as a trend within the scientific field of mathematics education, which led the Brazilian DDM community to gather as a subfield within the Brazilian Society of Mathematics Education, from now on SBEM.

Created in the late 1980s, SBEM developed as a fruitful environment in which different theoretical perspectives meet. In addition, teachers who teach mathematics at different levels of education, and researchers who develop systematic investigations join in this environment, can interact, and mutually nourish, their reflections and their professional practices. This wide and diverse environment is a heritage that we want to preserve, as we consider it fruitful in the quest to face the immense challenges of mathematics education as a social practice and as a scientific field. Within the SBEM, we can interact with several schools of thought and in the narrower space of the Didactics of Mathematics Working Group (GT 14), we deepen the theoretical and methodological debate aiming at the constitution of an increasingly robust scientific field. The DDM takes as its object of study issues related to processes in which there is an intention to modify the relationship between human beings and mathematical knowledge. These processes are part of the object of educational sciences and more specifically of mathematics education. In this text, we try to trace a brief history of the DDM, discuss characteristics that seem essential in this trend and highlight some of its contributions to mathematics education and the sciences of education. The desire to dialogue with the community of mathematics educators led them to choose to lose precision in the expression of certain ideas to ensure intelligibility by researchers who do not adopt this trend.

BRIEF HISTORY OF DDM IN FRANCE AND BRAZIL

In June 1993, the French community of researchers in didactics of mathematics celebrated its 20th anniversary with a colloquium (Artigue et al, 1994) which, at the same time, marked the foundation of the Association pour la Recherche en Didactique des Mathématiques (ARDM) and honored two researchers whose role in building the foundations of this research community is central: Guy Brousseau and Gérard Vergnaud. In this colloquium, a "scientific balance of the work carried out, of the themes, problems and methods used, of the constructed results" (Rouchier, 1994, p. 13) is carried out. Among the conferences, in addition to the two honorees (Brousseau, 1994; Vergnaud, 1994), there are internal views on the trajectory covered (Perrin-Glorian, 1994; Rouchier, 1994) and views from the perspective of other research communities in the field of mathematics education – from Italy (Boero, 1994), Switzerland (Brun, 1994), the United States (Kilpatrick, 1994) and Germany (Strässer, 1994). The communications are structured in seven themes: Cognitive approaches: schemas, conceptual fields; Theory of didactic situations; The teacher in the didactic system; Anthropological approach; Epistemological aspects and mathematical contents; Interactive learning environments with the computer; Methodological aspects of the research.

Taking a look at this historical moment seems opportune to us because aspects of the constitution of this community of researchers that preceded its explicit assumption as an institution are exposed and reflected about. At the same time, the traces of external gazes help to understand the specifics of DDM in the broader scope of mathematics education.

We highlight the central place of the theory of conceptual fields (TCF) and the theory of didactic situations (TDS) in the origin of DDM. We also observed the importance of the anthropological theory of the didactic (ATD) and the sensitivity to issues that concern the use of technologies in math learning and teaching as aspects that were considered important.

The project to establish the DDM in France as a scientific field is marked by affiliations and ruptures to entities whose goal is the mathematics teaching and the training of teachers who teach mathematics, such as Instituts de Recherche sur l'Enseignement des Mathematics (IREM) and the Association des Professeurs de Mathematics pour l'Enseignement Public (APMEP). We highlight the relationship with the IREM. With the worldwide advent of modern mathematics, between the late 1960s and the first half of the 1970s, the IREMs were created and spread across the different regions of France. Structures that bring together teachers from all stages from early childhood education to higher education, the IREMs are dedicated to carrying out pedagogical experiments related to math teaching and learning and the production of didactic-pedagogical materials and projects for initial and continuing education of mathematics teachers. The active and intense participation of teachers working at all stages of schooling in the IREMs between the 1960s and 1990s was favored by conditions such as reduced class hours, for example. Little by little, a network of IREMs was formed, with inter-IREM thematic commissions, organization of colloquiums, summer schools, and various publications.

At the heart of this complex relationship between communities that in different ways deal with issues of learning, teaching, and training of teachers who teach mathematics are the connections between mathematics education as a scientific field and as a social practice. The DDM arises from the understanding that the scientific fields consolidated at the time (such as mathematics and educational sciences, among others) were not able to adequately investigate the phenomena surrounding the learning and teaching of mathematics. At the same time, there was dissatisfaction with the approach to mathematics teaching issues from an innovation point of view, prevalent at the IREMs at the time.

Thus, throughout the 1970s and 1980s, research laboratories began to be founded, the journal RDM – Recherches en Didactique des Mathématiques was created, national didactic seminars were held regularly, and the Summer Schools of Didactics of Mathematics, which we will discuss later, took place biannually.

An important complement to the outline of this brief history of the DDM is extracted from the publication of the French commission for the mathematics teaching (Artigue; Trouche, 2016) produced in preparation for a presentation of the French didactic tradition at the 2016 International Congress on Mathematical Education (ICME). The organizers of this brochure chose to highlight scientific collaborations with countries in Africa (Benin, Mali, Senegal, and Tunisia), Asia (Vietnam) and Latin America (Chile, Mexico, and Brazil) and asked pairs of researchers from France and the countries in focus for a survey of this history of collaboration as part of a broader intention to rescue the memory of the development of this research tradition outside France.

This publication shows that the first interactions with other countries took place in the 1970s and were intensified over the years, especially from the 1990s, which coincides with the period of the creation of the ARDM. From now on, we will focus on the case of Brazil.

The first collaborations of the new-born French tradition of mathematics education with Brazil date back to the 1970s, with two groups: the Group of Studies on Mathematics Teaching of Porto Alegre (GEEMPA) and the Group of Studies and Research in Mathematics Education (GEPEM). GEEMPA was created by Professor Esther Pilar Grossi, who carried out her doctorate under the supervision of Gérard Vergnaud and GEPEM was created in Rio de Janeiro by Professor Maria Laura Mousinho Leite Lopes, who developed a partnership with IREM in Strasbourg.

Throughout the 1980s, researchers such as Guy Brousseau, Gérard Vergnaud, Michèle Artigue, and Colette Laborde made scientific visits to Brazil and conditions were created to strengthen cooperation ties between the two countries.

As of the 1990s, an important impetus was given through the establishment of international cooperation agreements financed by the Coordination for the Improvement of Higher Education Personnel (CAPES) in Brazil and by the French Committee for the Evaluation of University Cooperation with Brazil (COFECUB) in France. Since then, five agreements have been signed in which, over the years, several Brazilian institutions have participated: the Pontifical Catholic University of São Paulo and Rio de Janeiro (PUC-SP and PUC-RJ, respectively), the Federal University of Pernambuco (UFPE), the Federal University of Santa Catarina (UFSC), the Bandeirante University of São Paulo (UNIBAN) and the Federal University of Mato Grosso do Sul (UFMS). Sometimes as part of these agreements, sometimes through grants from CAPES and the National Council for Scientific and Technological Development (CNPq), between the 1990s and 2010s, according to Campos and Trgalovà (2016), 25 doctors (full doctorate in France, sandwich doctorate or co-supervision of a French researcher). These doctors were linked to more than ten universities in different regions of the country.

Little by little, a community of researchers in mathematics education dedicated to the DDM trend was formed in Brazil. Campos and Trgalovà (2016) show that in at least eight states, public or private higher education institutions were developing research using DDM theories. More than 400 master's dissertations and approximately 130 theses had already been defended at the time, adopting theories linked to the DDM. The authors also highlight joint publications between Brazilian and French researchers, scientific visits, and post-doctoral internships.

From an institutional point of view, it is possible to identify basically two ways of life of the DDM in Brazil: one is via higher education institutions, more specifically in the postgraduate programs, and the other is at SBEM. Let us start with the latter.

SBEM is made up of different working groups (GT, in the Portuguese acronym), created in 2000 in the 1st International Symposium on Research in Mathematics Education - SIPEM. In 2014, a group of researchers, members of SBEM, requested the creation of the Didactics of Mathematics Working Group. This research group considered necessary to create a space for discussion on issues specifically related to the theoretical field of DDM. GT 14 - Didactics of Mathematics, created in 2015, currently has about 60 participants, and since its creation, has developed actions aimed at broadening and deepening the debate on theories and research methodologies mobilized by its members.

From the point of view of the postgraduate programs (PPGs), there was also a great expansion of DDM in Brazil, with guiding professors working on this theoretical trend in several PPGs, contributing to the strengthening of Brazilian research and postgraduate studies.

The Schools of High Studies (Escolas de Altos Estudos - EAE), financed by CAPES, also brought many contributions to the development of DDM in Brazil. From 2008 to 2011, the EAEs were held at UNIBAN with Guy Brousseau, Gérard Vergnaud, Michèle Artigue, and Yves Chevallard. In 2015, Luc Trouche taught an EAE at PPGEDUMATEC, at UFPE, that was transmitted online to participants from other states such as São Paulo, Bahia, and Mato Grosso do Sul. In 2019, PPGEdumat/UFMS promoted an EAE on the ATD, involving eleven PPGs from the five Brazilian regions, with Annie Bessot, Avenilde Romo, Corine Castela, and Hamid Chaachoua as guests. About 200 math educators participated in this EAE, most of them master's and doctoral students.

Another important action that contributed to the strengthening of DDM in Brazil was the creation of the Simpósio Latino-americano de Didática da Matemática (LADiMa) in 2016. LADiMa, now in its third edition, was born at a meeting of Latin American researchers, participants in a summer school in didactics of mathematics in France. The proposal of this group was to create, in Latin America, an event with characteristics similar to summer schools in DDM.

> A summer school, in the sense that we have understood it since 1980, is neither a congress nor a school for young researchers. If we insist on the term school, it is because it indicates a commitment to study, a transmission of knowledge that does not belong to its authors, but which are, on this occasion, decontextualized, depersonalized, detemporalized, which, therefore, enter into a process of didactic transposition. (Margolinas et al., 2001, p. 11)

LADiMa is, therefore, a space for collective study built by and for the community of Latin American researchers in didactics of mathematics, which reaffirms its commitment to the study, sharing, and processes of decontextualization, depersonalization, and de-temporization that mark the didactic transposition of the knowledge produced by this community. Such study spaces, especially aimed at researchers in the area, have favored the consolidation of DDM both in France and Brazil.

The community of Brazilian researchers in the DDM field has grown significantly in the last two decades due to the multiplier effect of the qualification of PhDs in the different PPGs. Currently, about ten research groups develop investigations in light of the DDM, spread across the five Brazilian regions. As an example of the growth of Brazilian research in DDM, we bring a particular case of one of the theories that make up this field, the anthropological theory of the didactic. In a study on the development of the ATD in Brazil, Bittar and Bellemain (2021) identified 150 Brazilian dissertations and theses that mobilize this theoretical framework (sometimes articulated with other theories linked or not to DDM). Of these, 113 were carried out in the last decade, many of them guided by researchers trained in previous decades in Brazil.

Besides the research and teaching actions developed in higher education, many activities are conducted by researchers in DDM with teachers of basic education, linked to university extension. The Brazilian Constitution of 1988 and the law of guidelines and bases of national education (LDBEN 9.394/1996) established the principle of inseparability between teaching, research, and university extension. Through teacher education projects and the elaboration of resources or didactic sequences, a closer connection is settled between the university and society, providing that research results can nourish teaching practices, and, on the other hand, classroom experiences can enrich the research and teaching. This connection via university extension also increases the chances that the knowledge generated in universities responds to the needs of Brazilian society.

Anchored in this brief history, the next topic problematizes the process of emancipation of the DDM in relation to linked communities.

EMANCIPATION OF DDM

A new scientific field is always born out of a need. With DDM it was no different. In the 1950s and 1970s, the modern mathematics movement, the creation of the IREM, and the failure of the reform implemented in France created the conditions for the emergence of the DDM.

> As is often the case, the birth of this field was made in opposition to specific currents of thought or research and in accordance with others. The francophone paradigm of mathematics teaching still bears the mark. (Margolinas, 2004, p. 3)

Some characteristics are pointed out by this author. The DDM was born driven by the project of constitution as a scientific field, with a strong approach to the mathematicians' community, initially focused on the study of teaching processes in formal school spaces.

> The failure of the reform showed as early as the 1970s that other determinants were at work in mathematics teaching than cognitive and mathematics as separate entities. However, it also made French mathematics educators very wary of rapid classroom application and direct intervention in the education system. (Margolinas, 2005, p. 343)

Over time, DDM has developed rooted in related fields (mathematics, cognitive psychology, educational sciences, etc.) but also developed its own theories and methodologies to investigate phenomena connected to its object of study.

> One of the originalities of the French research paradigm in the didactics of

> mathematics is that it takes fundamental research seriously instead of student success straight away. It is about investigating conditions that theoretically make it possible to evolve students' knowledge and improve teaching. (Margolinas, 2005, p. 343)

It is also worth mentioning here the emancipation of the DDM in relation to the sciences of education and mathematics. It is interesting to note that parallel to this movement of DDM in France, in the United States, Shulman (1986) and collaborators discussed what they called the "lost paradigm" when analyzing teacher education in that country. For Shulman, in the studies carried out at the time, the content was practically left out; the research did not consider the specificity of the discipline, focused on procedures and "teaching effectiveness". Thus, Shulman's and DDM's concerns somewhat agree about the importance attributed to the questioning of the object of knowledge in research on teaching and learning.

As DDM grew and strengthened as a scientific field, new questions could be investigated, and its focus of interest broadened and adjusted. For example, if initially the DDM was mainly concerned with mathematics teaching and learning in a school situation, Bosch and Chevallard's (1999, p. 79) definition illustrates the detachment, for over 20 years, from the almost exclusive look at this context: « [...] la didacticique des mathématiques [is] la science de l'étude et de l'aide à l'étude des (questions de) mathématiques. »

Today, DDM aggregates several theories and methodologies. Without trying to be exhaustive, we propose to draw some lines in the construction of this complex that is the DDM.

There is a sector within DDM that is rooted in an emphasis on the cognitive processes involved in studying and helping to study mathematics. At the origin of this sector, in addition to the TCF, we highlight the theory of registers of semiotic representation - TRSR (Duval, 2009) and the instrumental genesis (Rabardel, 1995).

TCF is anchored in Piagetian epistemology and resignifies it in connection with Vygotsky's psychology and considering the specificities of mathematical objects. Through TCF, we can understand affiliations and ruptures in the long-term process of building mathematical knowledge. Many researches around the world, including Brazil, are based on the TCF to investigate issues of mathematics learning and teaching. The TRSR, in turn, places the importance of language in the process of attributing meaning to mathematics at the heart of the investigation. Giving sequence to those theories, in the same cognitive bias, the instrumental genesis (Rabardel, 1995) allows investigating the influence of artifacts in the process of construction of mathematical knowledge.

A second sector is initially constituted around the relationship between mathematics teaching and learning. In addition to the TDS, we place the theory of tool-object dialectic and interplay between frameworks. (Douady, 1986). These theories look at the didactic triangle (teacher, student, knowledge) and bring invaluable contributions to the study of conditions that may favor the learning of mathematical knowledge in a school situation. The development of both was methodologically supported by classical didactic engineering (Artigue, 1988; 2014).

In the early days of DDM, the focus was primarily on theorizing the relationship between students and mathematical knowledge and on the study of conditions that favored the students' attribution of meaning to mathematical knowledge. This look made it possible to break with the vision of a passive student who digests the knowledge that is masterfully exposed to him by a teacher. However, for a long time, the need to systematically study the teacher has been leading to expansions and reformulations in theories, as well as the constitution of new theoretical frameworks. This concern appeared in the Colloquium of the twenty years of the DDM (Artigue et al., 1994). Studies within the TDS and the ATD developed theoretical constructs through which to investigate the place and functions of teachers. The dual didactic and ergonomic approach, around Aline Robert and Janine Rogalski (Robert, 2008), focuses on the study of ordinary teaching practice. Continuing the TCF, studies along the lines of professional didactics emerged (Pastré, Mayen, Vergnaud, 2006). Rooted in the ATD and the TDS, the theory of joint action in didactics is born (Sensevy, 2011) and, from the perspective of instrumental genesis, we have the documentational approach to didactics (Gueudet; Trouche, 2010).

As we have tried to indicate, DDM is not a theory. Around this trend are grouped theories that bring different and -in our understanding- complementary contributions to the investigation of phenomena around the study and help for the study of mathematics, regardless of a school environment.

The theory of conceptual fields (Vergnaud, 1991, 1994) is situated as a cognitivist theory and not exactly didactic, but in addition to its undeniable importance for the historical emergence of DDM, it brings great contributions to a deeper understanding of the formation of mathematical concepts with strong implications for mathematics teaching.

The theory of didactic situations (Brousseau, 1997a, 1997b) put in a central place the problematization of mathematical knowledge, whose learning is aimed.

> we could no longer hold to the tepid creed of pedagogy in-so-far as it turned knowledge into a "false protagonist" of the play. Much to the contrary, we considered knowledge to be the main character and the true hero of the plot. (Chevallard, 2007, p. 131-132)

Considering knowledge (mathematical) a central (not peripheral) factor to investigate phenomena about the learning and teaching of this subject is one of the characteristics of DDM (not just from the TDS) that breaks with traditional didactics. The TDS establishes a strong relationship between knowledge and situation, the basis of the DDM.

> That knowledge was more or less the explanation of the pupil's behaviour was carefully elaborated by Guy Brousseau in terms of *didactic situations* – a magnetic concept around which French didactics still revolves (Brousseau, 1998). Knowledge is potentially encapsulated in situations, and it is in going through those situations that the pupil, or whoever, can learn. (Chevallard, 2007, p. 132)

Understanding the importance of empirical verification of theoretical formulations under deontologically responsible conditions led to Brousseau to create in 1973 the Center of Observations and Research for Mathematics Teaching (COREM)[1], integrated into the Jules Michelet primary school. In partnership with

[1] COREM operated from 1973 to 1999 and much very rich material was produced. Much of the collection is available at http://www.imac.uji.es/ CRDM

schoolteachers and other researchers, proposals were made for the teaching of mathematical content: design, a priori analysis, teaching and observation and, finally, a posteriori analysis of the experimentation carried out. This entire process was developed from the perspective of the TDS, which Brousseau called didactic engineering. In Brousseau (2013), the author describes the trajectory of creation of this research methodology, showing its intertwining with the theory of didactic situations, as is also evidenced by Barquero and Bosch (2015, p. 259):

> In the research program set up by the TDS, the experimental work carried out by DE processes is crucial, as it represents a way to empirically test epistemological and didactic proposals formulated in terms of sequences of adidactic and didactic situations.

A situation, according to the TDS, is a theoretical model of the interaction of a student (or students) with a medium designed with the intention of producing knowledge with as much autonomy as possible. The elaboration of situations in the light of this framework includes an epistemological and didactic study of the mathematical object at stake to think about alternative proposals to what the researcher (or teacher) considers not satisfactory.

> This ambitious project requires a double rupture: researchers need to allow themselves to question mathematics as it is usually conceived and presented by mathematics scholars and by school institutions, elaborating their own alternative reconstructions of mathematical knowledge and activities (the reference epistemological models). They also need to have the same attitude towards other disciplines (psychology, pedagogy, sociology, etc.) concerning the effects of their proposals on mathematical practices and knowledge. This is why it is important that the results obtained are empirically based, protecting researchers from adopting unfounded ideologies or implicit institutional viewpoints on both educational facts and mathematical knowledge. (Barquero; Bosch, 2015, p. 260)

With the support of the epistemological and didactic study carried out in the preliminary phase, the a priori analysis and the design of the didactic sequence are carried out. At this stage, we seek answers to questions such as: what activities should we propose? What are the variables of the situation? What strategies can students mobilize to solve the activity? At this stage of the investigation, we must make choices based on the research question and the objective to be achieved.

> Conception and a priori analysis is a crucial phase of the methodology. It relies on the preliminary analyses carried out and is the place where research hypotheses are made explicit and engaged in the conception of didactical situations, where theoretical constructs are put to the test. Conception requires a number of choices and these situate at different levels. (Article, 2014, p. 474)

To better understand the role that the a priori analysis has played over the years in DDM research, we will revisit the connection between the TDS and didactic engineering, as both were born interconnected.

> The situations must take into account, at the same time, the organization of mathematics, students' learning opportunities, and teachers' teaching conditions. Those situations are models of how mathematics works in teaching conditions. It is the theory that allows for the a priori analysis of situations, and it is the realization of didactic engineering that confronts this theory with contingency. (Perrin-Glorian; Bellemain, 2019, p. 48)

These authors show how didactic engineering has supported other theories, such as the theory of tool-object dialectic and interplay between frameworks (Douady, 1986) and the theory of conceptual fields (Vergnaud, 1991). Research has been carried out with these and other theoretical references mobilizing didactic engineering as a research methodology. The a priori analysis is a tool that has evolved over time along with this methodology and that has been used outside the execution of a didactic engineering, or the TDS.

Finally, it is important to highlight the role of the theory of didactic transposition (Chevallard, 1991) in the emancipation of the DDM, which teaches us that knowledge is not static, homogeneous, unquestionable:

> Knowledge is not a given, the theory says, it is built up, and transformed, and – such was the keyword – *transposed*. […] The main point in the didactic transposition theory is that it considers knowledge as a changing reality, which adapts to its institutional habitat where it occupies a more or less narrow niche. (Chevallard, 2007, p. 132)

Be it consciously and voluntarily or not, when an object of knowledge that exists in some scope migrates to an institution that aims to teach it, a process of didactic transposition takes place, adapting to the specific conditions and restrictions of this institution. For example, the object of knowledge area, before getting into school, already existed in other areas of social life. But for it to be learned and taught at school, a process of deconstruction and reconstruction of this object is necessary so that it adapts to the characteristics of the different years of schooling.

After this brief history and considering the complexity and diversity of the theories that make up the DDM, we chose to discuss, in the next topics, examples that illustrate relevant aspects and contributions of the DDM. We invite the reader to deepen the study of the themes briefly presented below.

THE TEACHER'S PLACE IN THE STRUCTURING OF THE ENVIRONMENT

For Brousseau (1997a, 1997b), the (didactic) situation is a model of interaction between a subject and an environment prepared by the teacher. For learning to occur, the environment must be challenging and provide the subject (the students) with feedback that leads them to gradually approach the mathematical object under study. Thus, as the students' state of knowledge evolves, the environment also changes, generating new layers in the therefore dynamic situation.

In the model initially structured by Brousseau, the students occupy five positions, characterized by the means with which they interact: material means, objective means, reference means, learning means, and didactic means. The subject-environment interactions of a situation constitute the milieu of the next layer. The teacher, on the other hand, occupies only two positions – preparing the class and giving the class – which signals a secondary role in the theorizing initially proposed. For Margolinas (2004, p. 12)

> The role assigned to the teacher in didactic engineering has hampered their emergence as actors in the didactic situation.

The teacher's role is highlighted in the devolution and institutionalization processes. On the other hand, in the adidactic situations of action, formulation, and validation, it may seem that the teacher leaves the scene. However, as the research progresses, the importance of studying the role of the teacher in situations becomes evident:

> The article by Gilbert Arsac and Michel Mante (1989) will be decisive, as it calls into question the idea that the teacher's role could be described quite simply or even minimized. They describe the great complexity of taking the teacher's role into account. (Margolinas, 2004, p.22)

Margolinas (2004) discusses the phases of conclusion, called evaluation and validation, that allowed him to “start to problematize the teacher's role”. Considering that in the model proposed by Brousseau the teacher's position is little approached, the author resumes the model and proposes some changes, reaching, over time, the proposal summarized in Chart 1, in which the different milieus (Mi), the positions that students occupy (Ei), the positions that teachers occupy (Pi) and the respective situations (Si) are represented.

Figure 1: structuring the environment

M+3 construção		P+3 noosférico	S+3 noosferiana
M+2 projeto		P+2 construtor	S+2 construção
M+1 didático	E+1 reflexivo	P+1 projetor	S+1 projeto
M0 aprendizagem	E0 aluno	P0 professor	S0 didática
M–1 referência	E–1 aprendiz	P–1 observador	S–1 adidatica de aprendizagem
M–2 objetivo	E–2 agindo		S–2 referência
M–3 material	E–3 objetivo		S–3 objetiva

Source: adapted from Margolinas (2004)

When we make an ascending analysis of the elements of this chart, we observe that the positions proposed by Brousseau regarding the student are preserved. However, for the teacher, only positions P0 (lecturing) and P+1 (preparing the class) were foreseen. In a global analysis of the situation, taking into account the teacher's actions not only with regard only to the class being taught, Margolinas (2004) inserts three more levels in the model: P+3, which corresponds to the most global level of the teacher's teaching project, such as their conceptions about teaching mathematics and the official guidelines for the subject; P+2, corresponds to the project of a chapter or a topic to be taught; and P–1, which corresponds to the moment in which the students are working and the teacher observes the situation. This author thus inserts a top-down analysis of the teacher’s role that allows a deeper understanding of their actions and even their learning process, since “The teacher, like everyone else, interacts with an environment, and he learns in this interaction, both consuming and producing knowledge.” (Margolinas, 2004, p. 71).

To better understand the study of teacher activity from the model proposed by Margolinas, we bring Figure 2 with the specifications of the levels of teacher activity.

Figure 2: Levels of teacher's activity

- +3 Values and conceptions about learning and teaching
  - Educational project: educational values, conceptions of learning, conceptions of teaching
- +2 The global didactic project
  - The global didactic project, of which the planned sequence of lessons is a part: notions to study and knowledge to acquire
- +1 The local didactic project
  - The specific didactic project in the planned sequence of lessons: objectives, organization of work
- 0 Didactic action
  - Interactions with pupils, decisions during action
- -1 Observation of pupils’ activity
  - Perception of pupils’ activity, regulation of pupils’ work

Source: Margolinas et al., (2005, p. 207)

It is important to observe the interrelationship between the levels in the teacher's action. When he is acting on one level, other levels interfere with his/her action. For example, when preparing his/her class (N+1), elements of level +3 and level +2 are strongly present. The teacher also takes into account the class to which this class will be given (N0). At levels +3 and +2, conditions and restrictions outside the classroom influence the teacher's decisions when preparing his/her class, while this preparation is also influenced by conditions and restrictions internal to the process arising from levels 0 and –1. Another important factor to be considered in the analysis of the teacher's activity is the issue of temporality. At each level of teacher activity, there are temporal components of the past, present, and future. When teacher are teaching a class (N0, present moment), they remember another class they gave on this same subject (N0, past moment) or a class they will give (N0, future moment) and that they would like to change something. Thus, the teacher's situation is characterized by tension and interaction between levels, as evidenced by Margolinas:

> In their classroom activity (level 0), teachers can be caught between their past project that serves as a guide, but also as a restrictive framework (level +1, past) and their future project (level +1, future). Likewise, in their activity outside the classroom, for example, when preparing a class (level +1, present), they are influenced by the past construction they made of the mathematical theme (level +2, past) that they plan to teach; but this preparation activity may lead them to modify that construction and consider a new future one (level +2, future). (Margolinas, 2004, p. 75)

The structuring of the medium as proposed by Margolinas provides an analysis technique that allows considering the point of view of the students (bottom-up analysis) and the teacher (top-down analysis), actors in the didactic situation. Its articulation with other theoretical elements favors the understanding of the teacher's work. This is the case of the doctoral research by Neves (2022), which articulated levels of didactic co-determination (Chevallard, 2019b) and levels of teacher activity (Margolinas, 2004) to understand didactic decisions (Bonnat et al., 2020) of a teacher.

RUPTURE OF EDUCATIONAL PARADIGM

In the same line of development of the TDS with the expansion of the structuring model of the environment, the ATD allows for a deeper understanding of the relationships between what happens in the classroom and other areas in which the objects of knowledge are present. The foundations of this theory are people, institutions, and knowledge that are closely linked:

> Behind the persons, and the knowledge, there appeared the institutions, to be regarded on a par with the persons, in the light of a dialectic between persons and institutions. Persons are the makers of institutions which in turn are the makers of persons. (Chevallard, 2007, p. 132)

By participating in an institution, people become subjects of that institution and occupy positions in it. On the one hand, what makes up the institution are its subjects, so there is no institution without people. On the other hand, the institutions in which a person participates shape the relationships that person establishes with the knowledge that circulates in those institutions. For example, a group of students, say 8th graders, studying mathematics with their teacher constitute an institution with at least two positions: student and teacher.

The concept of institution is quite broad, also giving knowledge a broad meaning,

> didactics not only cares for the knowledge recognized as such by some authoritative institutions – e.g. the institutions of higher learning –, but it has to broaden its object of study, just because in the life of institutions, bodies of knowledge appear intricately linked, from the point of view of ecological analysis, with entities that some authorities would refuse to call knowledge, although we need to take them into account in order to explain the

> fate of “true” knowledge. (Chevallard, 2007, p. 133)

From this perspective, how to think about the teaching process and, consequently, that of learning? What paradigm meets what we have stated in this text so far? Chevallard (2019a) presents some existing paradigms over the centuries, to, then, propose a break with the current paradigm. Due to the purpose of this article and the limitation of pages, we chose to present only two of these paradigms: the paradigm of visiting works (PVW) and the paradigm of questioning the world (PQW).

In the PVW, knowledge is considered ready and finished works to be visited. In this paradigm, there is a tendency to monumentalize teaching, fixing it and compartmentalizing it into disciplines, ignoring the process of didactic transposition: mathematics appears as unique. In this paradigm, the teacher's task is to present the works so that students can get to know, admire, and reapply them in situations similar to those addressed by the teacher. Guided by the PVW, teaching will hardly lead students to appropriate the reasons for being of the works visited.

> A (mathematical) work is “visited” by a class under the supervision of the teacher as if it were a monument, even a masterpiece, that, however impudently, we are expected to revere and bow to. This leads to what I have called the “monumentalization” of the curriculum. Now when, to the contrary, we adopt the inherentist stance, things change almost completely. The first historical step in this direction was taken a number of decades ago when the French “modern” didacticians, following in the wake of Guy Brousseau’s pioneering work (1997), set to tackle the general basic problem of didactics: Given a work w, find a question Q the study of which will, if not generate, at least leads one to come across w, regarded as a key resource to arrive at an answer A to Q. Such was the first systematic and effective effort to “demonumentalize” the mathematics curriculum (Chevallard, 2019b, p. 99-100)

Breaking with the dominant paradigm of visiting works, Chevallard (2019a) proposes the paradigm questioning the world. In this new paradigm, one starts with questions whose answers are not known and the works are tools that help in the search for the answer to the initial question. To better understand this paradigm, we will use an example extracted from Chevallard (2019a), in which he considers the following initial question *Q_0: On the radio people are talking about “felt temperature”. What does this mean?* From this question, several others appear, called derived questions, and to answer them, we resort to works. We observe that, in this case, a work is visited with a specific purpose, to answer a question. Thus, there is an inversion in relation to what is done in the paradigm visiting the works. In questioning the world, one does not know in advance the answer to the question posed, nor the paths that will be followed until one reaches the desired answer.

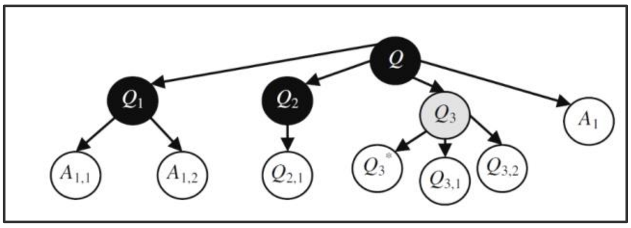
It is important to note the role of the issue in each of the two enunciated paradigms: in the PVW the answers are known, and the objective is to study the work, while in the PQW, the answer is not known. An example given by Marianna Bosch in a lecture at UFMS, which we believe illustrates the PQW well is the development of a doctoral research. A thesis must have a research question whose answer is unknown, otherwise it is not a thesis. To answer that question, the researcher must carry out studies, which raises other questions, paths are opened, and the researcher reaches an answer. Another researcher with another theoretical and methodological framework could follow other paths and arrive at another answer, which means that there is not just one answer to the initial question.

This example also illustrates a methodological tool developed at the ATD, from the perspective of the PQW, the study and research path (SRP) (Casabó, 2018; Chevallard, 2019b), which has been used in investigations in basic education, higher education, and teacher training. In the development of the SRP, Casabó (2018) defines some dialectics, among which we highlight the question-answer dialectic.

This dialectic allows drawing paths that can arise from the generative question (Q_0) until the desired response is obtained, called the heart response. This dialectic is represented by a map of questions and answers (MQA), and is presented as a methodology for the development of research in the ATD (Gonçalvez, 2022; Winslow et al., 2013).

Figure 3 brings an illustration of a MQA, where we can see that, from the initial question, three other questions and an answer are derived, and each of the questions generates answers or new questions. This process continues until a satisfactory answer to the initial question is obtained.

Figure 3: Example of question-answer map



Source: Winslow et al., (2013, p. 271).

In addition to the MQA being a very important methodological research tool from the perspective of the ATD, it also helps in the construction of the SRPs (CASABÓ, 2018) to be experienced by students (X) with the guidance of a teacher (Y) aiming at the study of a question (Q_0). It is important to emphasize that from the initial question, several paths can be followed, depending on the group that seeks to find the answer to the question, so, in this perspective, there is no correct or unique path.

FUNDAMENTAL DIMENSIONS OF A RESEARCH PROBLEM

In this section, we discuss what constitutes an investigation problem from a DDM perspective. We begin by distinguishing a teaching problem from an investigation (or didactic) problem. The first is the one that the teacher faces when having to teach a specific content.

> Teaching problems are formulated through the notions available in school culture, usually imported from curriculum documents (such as, for example, notions of motivation, meaningful learning, individualization of teaching, acquisition of a concept, abstraction, competence, etc.). The teaching problems are usually formulated, assuming and without questioning the notions and the dominant ideas in the school culture mentioned above. (Farras et al., 2013, p. 3)

It is important to point out that not every research problem in DDM arises from a teaching problem, as is the case of investigations related to the analysis of textbooks, for example. Another example is the study of factors that influence the decision-making of teachers in their teaching action.

But, how to characterize a research problem in DDM? This is a question that seems seminal to us and, of course, can have different answers, according to the theoretical perspective adopted. Gascón (2011) outlines an answer to this question, from the perspective of the DDM, more specifically, from the anthropological theory of the didactic, defining three dimensions of a didactic problem – epistemological, institutional, and ecological – described briefly below.

The epistemological dimension aims to describe and interpret the mathematical component of the research problem that results in the epistemological model of reference (EMR). Such a model is not unique, on the contrary, it has a provisional character:

> the epistemological models that the didactics of mathematics builds should be taken as *working hypotheses* and, as such, must be constantly contrasted y revised. (Gascón, 2011, p. 7)

The EMR, to be explained by the researcher, conditions the mathematical amplitude of the problem to be studied, the didactic phenomena visible to the researcher, the types of didactic problems that can be posed, the provisional and/or admissible solutions. In this way, the epistemological dimension of a didactic problem determines, or conditions, the other dimensions, having, therefore, a privileged position in relation to them.

> This EMR, of local or regional scope, must be compatible with the general epistemological model of mathematical activity which, in the case of the ATD, is formulated in terms of praxeological organizations or praxeologies. (Gascón, 2011, p. 9)

The economic-institutional dimension of a research problem includes questions related to the way things are placed in a given institution: how are things? What is the existing mathematical and didactic reality? To answer such questions, Gascón (2011) postulates that it is necessary to try to change what is stated through a clinical analysis.

The understanding of why things are the way they are and what conditions are necessary to think about alternatives to what is stated is the object of study of the ecological dimension of a research problem. Under what conditions and restrictions does a praxeology live, disappear, change, could it exist in a given institution?

> In principe, everything is a condition. However, we will say that a condition is a constraint on a specific U instance —a person or an institution— when U cannot, given all other prevailing conditions, reasonably expect, for a specific time, to be able to modify this condition. (Chevallard, 2011a, p. 12)

So, what is a restriction in an institution I_1 may not be for another institution I_2. For example, knowing the Kadiwéu language is a condition that cannot be modified to work with students who only know this language, so it is a restriction for teaching these students, but not for working with children who understand Portuguese.

Identifying and analyzing the conditions and restrictions that allow, favor, or prevent the dissemination of mathematical knowledge in a given institution is an essential task of the researcher in DDM.

Some of these conditions and restrictions are better known to teachers, such as conditions that their students need to meet to solve a specific task, material needed to carry out an activity. These are examples directly related to the subject matter, in the case of mathematics, but there are also factors that influence what happens in the classroom that come from other dimensions of reality. For example, a decision taken by a minister of education that implies a change in workload will directly affect the time devoted to a given topic. Or, to cite a current example, the Covid 19 pandemic implied drastic changes of various types in education and classroom practice. To study conditions and restrictions arising from different spheres, Chevallard

(2019b) presents the higher levels of didactic codeterminacy - humanity, civilization, society, school, and pedagogy - and lower – discipline (subject matter), sector, domain, theme, and questions. Each of the elements of this scale influences what happens in the classroom but also outside it. Figure 4 shows that the arrows between the levels are placed in both directions indicating comings and goings or reciprocal influences between the different levels.

Figure 4: scale of levels of didactic codeterminacy

Humanity
↑↓
Civilisations
↑↓
Societies
↑↓
Schools
↑↓
Pedagogies
↑↓
Disciplines ⇄ Sectors ⇄ Domains ⇄ Themes ⇄ Questions
↑↓
Didactic system $S(X, Y, \wp)$

Source: Casabó (2018, p, 4037)

The scale presented in Figure 4 corresponds to the PVW, as the lower levels, relative to the subject matter, are defined according to the formal curriculum, divided into subject matters that are subdivided into sectors, domains, themes, and questions. After this determination, we arrive at the didactic system. In the PQW there is a change in this schema. The lower levels start with the S(X, Y, Q) didactic system, which means that the study is guided by a question and, according to it, the rest develops.

The scale of codeterminacy levels has represented a rich tool for analysis, including research on teacher education that mobilizes other theoretical contributions, such as studies by Shulman, Tardif, and Schön (c.f. research developed by Neves, 2022). This tool allows you to study phenomena outside the classroom, such as the Covid 19 pandemic or the change of a government and its influences on a teacher's class (and student learning).

> This does not mean that the didactics must deal with all social, school, and pedagogical phenomena (which would be both absurd and impossible), but it must study the *didactic effects of these phenomena*, and decide how they affect the diffusion of mathematical praxeologies in a given institutional environment. (Bosch; Gascón, 2009, p. 97)

The study of restrictions and conditions coming from the different levels of the codeterminacy scale is also important both to understand teaching choices and to study conditions that allow the establishment of educational paradigms that meet an alternative proposal, like the one we presented in the previous section.

CONCLUDING REMARKS

We sought to show in this text that the tendency of mathematics education known as didactics of mathematics, was created in France almost half a century ago, and had great impulse from the formal establishment of a research association - the ARDM, in the 1990s. Its connection with the international community of mathematics educators, including Brazil, was established from the beginning, and intensified in the last thirty years.

Among the outstanding characteristics of the development of this trend, we highlight the valorization of investment in theoretical research, the continuous commitment to systematic study, in specific institutional spaces such as summer schools and Latin American symposia on didactics of mathematics, the importance attributed to problematization of mathematical objects whose learning and teaching are investigated, and the confrontation between theoretical constructions and reality under ethically responsible conditions.

We emphasize that, from its inception to the present, the scope of the objects of study of the DDM has been expanding. The DDM today aggregates a plurality of theories and methodologies that, from our point of view, nourish each other and complement each other in the search for a better understanding of the phenomena related to the intentional processes of diffusion and acquisition of mathematical knowledge.

Brazil currently has a large community of researchers in DDM in SBEM, a society that brings together researchers from the most diverse trends in mathematics education as well as teachers who teach mathematics at all stages (from early childhood education to graduate studies) and teaching modalities. This condition seems to us to favor the intertwining of theories from the field of education with theories from the DDM, present in many studies developed in Brazilian PPGs.

As we said in the introduction to this text, we see school education as essential for the autonomy of the citizen and from this perspective, we believe it is fundamental to investigate the mathematical activities that take place in schools, including the democratization of access to hegemonic mathematics, relating them to elements outside the classroom.

We then chose examples that seem to briefly illustrate the expansion of objects studied in the DDM, the development of theoretical-methodological tools and the articulated look at the mathematics at stake and the other dimensions of didactic phenomena.

At the TDS, initially, the focus on generating conditions for the students' development of an autonomous mathematical thinking caused the study of the teacher's role to be placed in the background. Later, this role began to be investigated, leading to the enrichment of the theoretical model, which establishes strong connections between the classroom and aspects external to its time and space.

Likewise, from the perspective of the ATD, the levels of codeterminacy, the discussion about didactic paradigms, and the dimensions of a didactic problem allow us to understand mutual influences between the teaching of specific mathematical contents, school education, and life in society, among others.

We hope this text and the numerous references that follow will help the researcher who does not carry out investigations from the same theoretical perspective as us to feel invited to study the theories we have outlined.

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