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ARTIGO ORIGINAL/ ORIGINAL ARTICLE

Respecting learner´s knowledge: evaluation through self-assessment

Respeitando o conhecimento do aluno: avaliação por meio da auto-avaliação

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ABSTRACT
In civil society movements for justice, one aspect of respect means people being central to their own liberation. Joao Pedro Stedile, a member of the the Brazilian Movimento dos Trabalhadores Rurais Sem Terra national board, reflects that one of the most significant aspects of their work is to "[win] back the worth and dignity of the peasant." (as cited in Peters & Podur, 2002). Even those without formal education, even people who are illiterate, can engage actively in knowledge creation to improve the conditions of our world. As Paulo Freire argues, knowledge is continually created and re-created as people reflect and act on the world. I argue that involving students in evaluating their own knowledge, dialoguing with teachers, and studying together with others, can contribute to both increasing students' self-respect, as well as deepening their knowledge of mathematics. I review the competencies and evaluation methodologies underlying Quantitative Reasoning at The College of Public and Community Service. Then, I focus in detail on our Criticalmathematical Literacy Self-Assessment.

Keywords: Criticalmathematical Literacy Self-Assessment, Quantitative Reasoning, self-respect.

RESUMO
Nos movimentos da sociedade civil por justiça, um aspecto do respeito significa que as pessoas são centrais para sua própria libertação. João Pedro Stedile, membro do conselho nacional do Movimento dos Trabalhadores Rurais Sem Terra, reflete que um dos aspectos mais significativos de seu trabalho é "[conquistar] o valor e a dignidade do camponês" (como citado em Peters & Podur, 2002). Mesmo aqueles sem educação formal, mesmo pessoas analfabetas, podem se engajar ativamente na criação de conhecimento para melhorar as condições de nosso mundo. Como argumenta Paulo Freire, o conhecimento é continuamente criado e recriado à medida que as pessoas refletem e agem sobre o mundo. Argumento que envolver os alunos na avaliação de seus próprios conhecimentos, dialogar com os professores e estudar em conjunto com os outros pode contribuir tanto para aumentar o auto-respeito dos alunos quanto para aprofundar seus conhecimentos em matemática. Reviso as competências e metodologias de avaliação subjacentes ao Raciocínio Quantitativo na Faculdade de Serviço Público e Comunitário. Em seguida, concentro-me em detalhes em nossa Autoavaliação da Alfabetização Matemática Crítica.

Palavras-chave: Autoavaliação da alfabetização matemáticacrítica, raciocinio quantitativo, autorrespeito.

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INTRODUCTION

The first time I visited an encampment of the the Brazilian Movimento dos Trabalhadores Rurais Sem Terra (MST, Landless Rural Worker's Movement), over 100 people, of all ages, came to ask me questions. They were living in very rudimentary shelters, as they were occupying the land and in the midst of a struggle to gain ownership. My dear friend/colleague/comrade, Brazilian Professor Gelsa Knijnik, whose political and intellectual work is with the MST, told me that most of the people there could not read or write. The first question a young man asked me was why is the United States blockading Cuba? Then a middle-aged person asked me a technical question about Monsanto's genetically engineered seeds. These people are "illiterate" in terms of reading and writing, but they have not been de-educated, or "processed" as Bob Moses might say[1], and they have access to information and ideas through radio stations that present international news from the perspective of those outside the circles of power.

In civil society movements for justice, an important aspect of respect means people being central to their own liberation.[2] João Pedro Stédile, a member national board, reflects that one of the most significant aspects of their work is that:

> We've won back the worth and dignity of the peasant. That has immeasurable value. It doesn't show up in statistics. But when a person stops being humiliated, stops being a slave, and they can walk with their head up, master of their own future, that's the most important thing we're building. (as cited in Peters & Podur, 2002)

Walking that way, even those without formal education, even people who are illiterate, can engage actively in knowledge creation to improve the conditions of our world. As Brazilian educator Paulo Freire argues, knowledge is continually created and re-created as people reflect and act on the world. Knowledge, therefore, is not fixed permanently in the abstract properties of objects, but is a process where gaining existing knowledge and producing new knowledge are "two moments in the same cycle" (Freire, 1982). Knowledge does not exist apart from human consciousness; it is produced by us collectively searching and trying to make sense of our world.

Further, Freire's ideas about problem-posing are intended to involve the learners in dialogue and co-investigation with the teachers. Freire insists that people cannot learn through "banking" where expert teachers deposit knowledge in the presumably blank minds of their students, who memorize the required rules in order to get future dividends. He stresses that this dialogue does not involve teachers'

[1] Bob Moses respected his teacher Ella Baker's concept of leadership, which highlights how leaders emerge from the community, in ways that people find their voice and make their own demands, helped in their growth by grassroots organizers. This process also underlines Moses' belief in the important intellectual understandings of people who are directly impacted by community issues, whether or not they have formal educational credentials. In Cortés' video tribute (2021), he quotes Bob describing Fannie Lou Hamer and others coming out of SNCC: "What they knew about was life. They had not been processed yet."

[2] Baldwin (1963) focuses on the importance of self-respect and of teachers instilling this in students through teaching the truth about our world. Further, Baldwin argues that with self-respect, Black children will not accept the conditions of our unjust world: "… one of [their] weapons for refusing to make peace with it and for destroying it depends on what [they] decide [they are] worth. … there are currently very few standards in this country which are worth a [person's] respect. [I would teach that] it is up to [them] to change these standards for the sake of the life and the health of the country."

pretending ignorance. Since no one is omniscient and people each have different experiences related to the themes under investigation, teachers and students can truly learn from each other. As Freire states: "Our task is not to teach students to think—they can already think; but, to exchange our ways of thinking with each other and look together for better ways of approaching the decodification of an object" (1982).

In this article, I will focus on alternative ways of evaluating students' mathematical

knowledge which actively involve the students evaluating their own knowledge, dialoguing with teachers, and studying together with others. I contend that this can contribute to both increasing students' self-respect, as well as deepening their knowledge of mathematics. I will first review how the mission of The College of Public and Community Service (CPCS, University of Massachusetts/Boston) fostered the development of seriously respectful curricula. Then I will outline the key competencies and evaluation methodologies underlying Quantitative Reasoning in our curriculum, including specific examples we studied that created a respectful learning environment. Finally, I present details of our Criticalmathematical Literacy Self-Assessment, concluding with a glimpse of other materials that involve students in a central role in their learning.

THE COLLEGE OF PUBLIC AND COMMUNITY SERVICE: AN OPPOSITIONAL PUBLIC EDUCATION INSTITUTION

In 1978, I answered an ad in *The New York Times* education section for a position at the College of Public and Community Service (CPCS), one of the colleges at the University of Massachusetts in Boston. I could not resist applying for a job in a school with that name. In spite of a less than warm Boston reception: after dropping me off for my interview, my ex-husband went to help someone lying in the middle of the heavily trafficked road in front of the school building, and got kicked at and screamed at by the person; and, in spite of my exiting from my interview through a door that set off a fire alarm, an excruciatingly piercing and long one, I knew, that day, this was where I belonged. I stayed almost 40 years. I stayed until, in essence, the college was closed by the central administration, our canary-in-the-mine-shaft struggle to remain being lost to the fact that " "social justice is just not popular anymore" [3] and the fake-austerity privatization of public higher education.

CPCS was an institution with oppositional values to mainstream institutional individualistic, competitive social relations. This was a place where about a third of the faculty and 10 percent of the staff belonged to our Socialist Collective. The flyer (n.d. but from around the late 1980's) in which we described ourselves for others who might wish to join, stated that we were:

> an activist group…who believe that our society has a number of problems which cannot be solved within the existing system. Chief among these are the divisions between different classes of people. This "primary contradiction" perpetuates the poverty, racism, sexism, and violence which we see all around us. The

[3] This is a quote from then-Provost Paul Fonteyn at a CPCS faculty meeting, sometime during the latter part of his position (2002-2008).

> network of public and community services can make such conditions more tolerable, but these programs cannot offer a lasting solution. In fact, these programs are often used to cover up the real causes of social problems and undercut opposition to injustice and inequality in our society. As activists we are committed to bringing out these contradictions in public and community services and also to promoting these same services as vehicles for progressive change wherever possible. At CPCS our work involves developing a socialist view of society, participating actively in struggles for democracy and equality, and politicizing the college community concerning the realities of both higher education and public and community service.

Significantly, to give a glimpse of the atmosphere at the college, "we were a group of people who were seen as influential within CPCS and the community—not as a radical fringe. That was rare, even then." (Withorn, 2016).

CPCS was the environment in which Socialist Collective comrade, labor historian Jim Green, helped me understand why I needed to write a significant theoretical article for tenure, and helped me understand how to write it, and thereby, gave me a strong foundation from which to apply for tenure. This was the institution in which one of our peer tutors stated he loved to work because, in contrast to his UMass/Boston College of Management tutees, when CPCS students came late to his group sessions, they took time to catch each other up. This was the place in which, at my department's introductory meeting with our new Dean, James Jennings, the first African-American/Puerto Rican Dean of our college, forgetting our new Dean was present, I lapsed into my Brooklynese during a passionate discussion of racism in educational institutions, and angrily stated that I had heard many NYC teachers speaking of their students as "fucking animals." I did not yet have tenure, and James would be the Dean when my case came up for review. Not a good lapse on my part… Fortunately, when I bumped into him in the elevator the next day, and apologized profusely how I did not mean to offend him, he said he appreciated my comment as he, too, had experienced the lack of respect and racist treatment many teachers exhibited towards their students. The respect at CPCS was real, going beyond words, even my colorful Brooklyn ones, to underlying meanings and social justice conceptions of community.

Our teaching/learning environment, as well as the curricular ideas we developed, grew out of that deep, real respect for all in our community, underpinned by our social justice education mission.[4] Our curriculum

[4] Our mission statement, written when we revised our curriculum in 2000, stated that: The College of Public and Community Service is one of five colleges that comprise the University of Massachusetts Boston campus. As Such, CPCS seeks to extend the tradition of the land grant university in a number of ways:

- By educating students to foster the public good and aid the transformation to a more equitable society
- By providing research, advocacy, technical assistance, and service to the surrounding community
- By forging partnerships with public agencies and community organizations that enhance the quality of life for low income and other inadequately served populations

In these ways, the college works toward overcoming the attitudes, beliefs, and structures in our society which prevent access to the resources that exist and discourage full participation in economic, civic, cultural, and political life. As an alternative educational institution, CPCS endeavors to function as an inclusive, democratic, and participatory learning community which promotes diversity, equality, and social justice. CPCS actively cultivates a diverse and mature student body and offers an empowering and effective education which equips students to advocate for themselves and to improve the health and well-being of their chosen communities. The college recognizes that, particularly in a multicultural society, such an educational enterprise is inextricably bound to the complementary goals of meaningful access and

connected theory and practice with the goal of aiding the transformation to a more equitable society, and insuring that our graduates demonstrated the critical consciousness needed to clarify and challenge prevailing values, ideologies, and practices. Our general education studies included Media Literacy, Public and Community Action, Understanding and Making Arguments, and Quantitative Reasoning. At CPCS, reading and writing were not just exercises students do for the teacher. We emphasized that these skills involve more than just learning to critically analyze others' ideas and to express your own ideas. The most important aspect of reading and writing is what Brazilian educator Paulo Freire would call "reading and re-writing the world." Outside our general education core, students could major in a variety of public and community areas—all with a focus of using theories and skills in the service of building a more just society: Community Media and Technology, Gerontology, Community Studies, Human Services, Community Planning, Legal Education, Labor Studies. Students could also minor in Youth Work, Management of Human Services, Community Organizing, and Educational Training and Development.

Many in our CPCS community had difficult life experiences and had been marginalized because of our race, gender, sexuality, age, or other aspects of our lives that are challenges to overcome in the context of an unjust world. When these challenging life experiences interfered with students' prior learning, we did not consider this as a "deficit," but rather as another resource in the educational process, a resource that contributed to our interweaving of learning and community activism. Our focus goes against so much thinking in education in the United States, and is in concert with the researchers described by Soviet cognitive psychologist A. N. Leont'ev. He emphasized the contrast between researchers in the US who are "seeking to discover how the child came to be what he [sic] is" with researchers in the Soviet Union "striving to discover not how the child came to be what he is, but how he can become what he not yet is." (Wertsch, 1985, p. 67) Nevertheless, some of our students had internalized the messages from the public schools they attended that they were academic 'failures,' and, consequently, they had not developed 'Standard' English reading and writing and quantitative skills. We appreciated the importance of intellectual diversity, teaching students those 'Standard' skills, and, at the same time, teaching the connections between knowledge and power that define certain skills as the 'Standard.' We wanted our students to have all the skills they needed to succeed as individuals, as well as all

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adequate support for underserved populations. The successful CPCS graduate is a competent, confident, self-directed, life-long learner who can demonstrate:

- The language and technical skills necessary for purposeful inquiry and communication
- The professional competence to function effectively in a broad range of workplace and community-based roles
- The critical consciousness needed to clarify and challenge prevailing values, ideologies, and practices
- The essential knowledge required for participating fully in society

The CPCS curriculum is designed with such students in mind. The core of this inventive educational system is a self-paced, competency-based, outcome-oriented curriculum in which prior learning is validated and collaborative projects are encouraged. At CPCS, the student is considered a resource in the educational process, and the acquisition of knowledge and skills intersects with experiential learning and field-based education. As a forward-looking educational institution, CPCS continues to explore innovative delivery systems and technologies and seeks to articulate its educational philosophy and pedagogy with other academic institutions, community organizations, and public agencies.

the skills and knowledges needed to fight for a better world for everyone. We do not confuse knowledge of reading and writing and math with knowledge of the world and of justice. We want our students to have all these knowledges.

The CPCS curriculum had various ways of connecting theory and practice, along with students working on real-life group projects. For examples: our Center for Immigrant and Refugee Community Leadership and Empowerment (CIRCLE) recruited and supported cohorts of newly immigrant students who complete a Community Leadership and Development Certificate, with subsequent opportunity to pursue their undergraduate degree. Women in Community Development (WICD) was a partnership with the Women's Institute for Housing and Economic Development and Project Hope, supporting low-income women of color to realize their educational goals. Working with CPCS faculty, students in the WICD cohort completed their studies while investigating the barriers faced by low-income people to acquiring higher education. WICD students presented their findings in local and national conferences and universities, and are earned academic credit while working with welfare rights organizations focusing on education and training policies. CPCS also had community classrooms where we delivered a portion of our curriculum to students at their community agency workplace, as a way of creating supportive pathways to higher education. Groups of youth workers at the New England Home for Little Wanderers, paraprofessionals at the Fletcher/Maynard Academy, and Wampanoag tribal members participated in these community cohorts. A sample of other projects through which students also earned competencies counting towards their degree requirements involved researching and writing a legal rights manual for teen girls in Massachusetts; preparing impact assessments and developing organizing strategies with the Coalition to Protect Chinatown; and, working with residents of the Dudley area of Roxbury to examine the effect of welfare reform and to advocate in the interests of the residents.

We were a dangerous institution. We were totally open about our social justice agenda. In 1990, a Boston Globe editorial described us as a "street-smart college." We managed to survive for almost 40 years. But, thinking critically—*really* thinking critically—is not a 'Standard' skill that is valued. In our increasingly mean-spirited, unethical world, many in the university central administration, and well-beyond, were not "comfortable" with our curriculum or with our students, or with our activist academic faculty and staff. The fact that we were also half women and a third people of color (faculty/staff, as well as students) added to their discomfort—we were not only too 'red,' but also too Black and Brown and Yellow and Red and White.

CRITICALMATHEMATICS LITERACY IN THE CPCS CURRICULUM

Clearly, CPCS was the perfect institutional environment for my development of criticalmathematical literacy: a community of social justice activists, respectful of all people; an explicit social justice mission, and educational practices that promoted a profound teaching and learning environment, all connected to actions for social change.

When our mostly adult students chose CPCS, they were mostly already working in public and community service fields; they had all, already, chosen not to major in mathematics or science or fields that required much mathematics. We designed a quantitative reasoning curriculum that

focused on skills and concepts needed to understand and make arguments in the fields of their majors, and in their role as citizens participating in our governmental and other institutions. Further, our competency-based CPCS curriculum was designed to shift the power balance between teachers and students. Students might take a course to demonstrate competence, but they also might do a real-world project or submit a report they wrote for a community agency that uses the skills delineated in the competency criteria. If a student felt one teacher was not evaluating their work fairly, they could negotiate with another teacher to complete their demonstration of the competency.

We wanted to have math literacy throughout the curriculum, so we conceived of teaching those skills as part of critical reasoning. Our required first level competency, "Understanding Arguments," focused on arguments where much of the evidence used to support claims was quantitative. Our required first level "Media Literacy" competency also included aspects that called for quantitative reasoning about how numerical information is and is not presented in the media. At the next level, we required a basic descriptive statistics—the range of quantitative skills that we judged was needed for active participation in community struggles for justice. We developed an optional third level competency that focused on descriptive and inferential statistics, but also had other competencies at Level 3 that included quantitative reasoning, such as "Economic Distribution" which required understanding quantitative information about such topics as income and wealth distribution, and tax policies. Each competency specified the rationale for *why* we included it in the curriculum; the precise statement of *what* the competency meant; the criteria for *how* to demonstrate the competency; the standards for *how well* you needed to demonstrate the competency; and, examples of *where* and *when* you could demonstrate the competency.

Further, our sequencing of competencies was designed to respect students' intellectual activity. The difference in the levels was not that the initial ones were "easy" and got progressively "harder." They were all challenging. The difference was that at the beginning levels, there was more faculty support; as the levels increased, students were in more independent learning environments. Throughout the curriculum, of course, there was significant interactions between teachers and students; but those interactions gradually, supportively, shifted the balance of learning from the teachers to the students.[5] This analysis is consistent with Soviet cognitive psychologist Lev Vygotsky's (1896-1934) theories of the development of young people's minds *through* social interaction. He defined a "zone of proximal development" as the gap between what a child currently understood on their own and what the child *could* understand in a

[5] I share with my students that my mother wrote all my papers for the first half term of my Freshman English class at Brooklyn College. I was completely bored with the readings and had no idea what to say about them. I was just going to college because it was completely tax-payer supported (i.e., free), my parents could afford to support me living at home, just working in the summers to pay for my textbooks, and my mother hoped I would graduate with an MRS (i.e., she hoped I'd marry a 'higher class' man than she had). She was not a teacher, but was determined to not let me fail, and I guess had the philosophy that this kind of cheating was a survival skill. So, she read all the selections and wrote the papers that I then copied into my neat handwriting. At some point, one of the readings interested me a bit, and as I was copying her paper, I realized that I did not agree with her ideas. I rewrote that paper, wrote all my future papers, and graduated summa cum laude.

supportive learning environment working with others who at that point understood *more*. (Vygotsky, 1978, p. 86)

For a specific example of how the levels worked, in evaluating our Level 1 Understanding Arguments competency, most of the assignments involved reading or listening to a complicated argument, identifying the main claim, the mostly quantitative evidence the author uses to support the claim, the reasoning that shows how the evidence supports the main claim, as well as identifying various counter-arguments, and how the language used clarifies or obscures the claim, and the biases of the author and the publication, the larger context and purpose in which this argument serves, and identifying the unstated, "taken-for-granted" assumptions in the argument. At this beginning learning level, I provided continuing opportunities for revision. Because the reading and other curricular materials were quite challenging, most students needed to re-read, re-study and revise their work. Their first submission was expected to be a thoughtful draft. After I commented on those papers, we discussed the arguments in class, answering student questions. Students were not allowed to take notes while we are talking about the articles, but encouraged to listen for understanding. We basically covered the answers to the competency assignment questions through a wide-ranging discussion that covered many additional related topics. Through re-studying the articles after our class discussion, and discussing them with others outside the class, seriously trying to understand more, students would see that the answers were already in the articles. Then students re-write their papers. If there are still large areas of misunderstanding, we again review those in class. Otherwise, I meet with any individuals who may need to revise again. The idea was if you understood after the class discussion, and reflected that understanding in your revision, you were learning, not "cheating."

THE QUANTITATIVE REASONING COMPETENCY STATEMENT[6]

Rationale: In all areas of public and community service, as well as in our roles as citizens and consumers, we are confronted with information in the form of quantitative data. This information may be included in news stories, journal articles, political pamphlets, or other media. It may be presented in the form of graphs, tables, or integrated into written text. Our actions as workers, citizens and consumers can often be strengthened through attention to quantitative reasoning. Further, we can often understand issues more deeply by gathering quantitative data that are not initially presented. In order to make informed decisions in our actions as workers, citizens, and consumers, we need to consider quantitative as well as qualitative information.

Competency: Can use numerical information to gain insight and understanding and draw conclusions about public and community issues.

Criteria: The student must demonstrate the ability to:

1. Understand and draw conclusions about public and community service issues when dealing with quantitative information

[6] The team of mathematics educators that developed this competency included me, as chair, and Fadia Harik, Joe Cooper, Sylvia Rosales, Manny Rosado Torres, and David Reider. Also, the late Michael Stone provided excellent feedback at a number of points in our work.

about those issues. In particular, you must be able to:

- understand and draw conclusions from short readings about the issues, where an argument is supported by quantitative evidence;
- verify the calculations in an argument about public and community service issues that relies on quantitative reasoning;
- understand how to read graphs or charts about such issues and know how to work with different kinds of charts and/or graphs in getting information about the situation summarized in those graphs and charts; and,
- understand and draw conclusions from quantitative information about public and community service issues presented in other media besides articles or graphs/charts, such as political cartoons or advertisements.

2. Use quantitative information about public and community service issues to construct arguments, drawing conclusions about those issues.

3. Present numerical information in effective ways.

Standards:

For all the Criteria, use of a calculator is encouraged.

1. In all the Criteria, you will be evaluated on the reasoning you use to get your answer, as well as the accuracy of your answer. This will involve demonstrating the ability to:
    a. interpret and estimate quantities, including very large and very small values;
    b. extract quantitative information from graphs, tables, news stories, journal articles, or other media;
    c. present quantitative information in graphical, tabular or other written forms;
    d. compare quantities using differences, ratios, rates, and percents;
    e. interpret and apply basic descriptive statistical concepts appearing in various media, including measures of central tendency (mean, median and mode) and the results of surveys; and,
    f. recognize and pose real world problems involving the use and/or collection of data.
2. In the readings referred to in Criterion (1a) you will be asked to briefly discuss the main point, how the numerical information supports (or does not support) that point, and what other quantitative data you could examine to further learn about that point. You may be asked to use the given numbers and your basic quantitative skills to gain more information about the situation (i.e., to create and solve some quantitative questions that are meaningful to the analysis of the argument). Further, you may be asked to solve a math problem that involves making reasonable assumptions about rounding the

data, choosing which given data are relevant, discussing which missing data would be relevant, and/or performing multiple operations with very large or very small numbers.

3. In the arguments referred to in Criterion (1b) you will be asked to show how each number in the argument was gotten from which previous data, or if some of the information is missing, you will be asked to state which information you would need and how you would go about verifying the calculations if that missing information were provided. Further, you will be asked to discuss possible sources from which you could find the missing information. You will also be asked to use the numerical and other evidence to evaluate the reasonableness of the argument.
4. In the graphs and charts referred to in Criteria (1c) you will be asked to explain and evaluate the kinds of numerical questions that are answered by the graphs and charts, and the kinds of numerical questions that cannot be answered, and why. You will also be asked to solve problems by gathering information from one chart or graph to operate on the information from the other graphs/charts. Further, you will be asked to describe what kind of raw data were used to construct the graphs and charts. Finally, you will be asked to draw conclusions from the information presented in the graphs and charts and to indicate what other kinds of numerical data you would want to clarify or deepen the conclusions that can be drawn from the given graphs and charts.
5. In the other media referred to in Criterion (1d) you will be asked to describe the main argument. You will be asked to explain how the numbers support (or do not support) the point of the argument. You will be asked to judge if the use of numbers in the presentation (picture, TV show, etc.) clarifies or obscures the point of the argument. You will be expected to reason quantitatively about at least two media other than prose or graphs and charts.
6. For Criterion (2), you will be expected to be able to use "raw" numerical data that you are given and also to be able to collect and/or research a small amount of "raw" numerical data that you can then use to construct an argument about a public and community service issue. You will be expected to detail how the numerical data support the argument, and discuss what kinds of data might be collected that would further strengthen or challenge your argument.
7. For Criterion (3), you will be expected both to criticize others' summaries of data, re-presenting the data in clearer graphs or charts, and also to present the data given to you or gathered by you in Criterion (2), in clear forms. By clear forms, we mean graphs or charts that summarize all the data in ways that make the main conclusions that can be drawn

from the data easy to understand and that do not mislead the reader to draw incorrect conclusions.

Examples of Demonstration

1. Prior Learning: If you feel confident of your quantitative reasoning skills and wish to demonstrate this competency based on your previous knowledge and experience, you first need to complete a quantitative reasoning self-assessment. Based on your performance in this diagnostic, a quantitative reasoning evaluator will recommend whether you may attempt to demonstrate the quantitative reasoning competency through prior learning. The evaluator will discuss with you why they reached their conclusion and you will have an opportunity to negotiate with a different evaluator if you disagree. A prior learning demonstration will be accomplished through the successful completion of two components:

a. a proctored test; and,
b. a response to an open-ended question that would demonstrate the depth and breadth of your knowledge in relation to the competency. This response may be completed on your own time and it may take one of three formats: a 2-4 page paper; a multimedia presentation with appropriate documentation; or, a portfolio of prior work.

Based on your performance, your quantitative reasoning evaluator will either certify completion of the competency demonstration or will specify additional work that you will need to complete through independent study.

2. Independent Learning: If you feel confident of your quantitative reasoning skills and feel that your ability to interpret quantitative data in different contexts is strong, but you need some review of the math, and you feel that you can learn the concepts on your own, you can meet with an evaluator to discuss this possibility. You go over the competency criteria together, identify possible sources of information and materials you might read, and emerge from the meeting agreeing to put together a written plan for acquiring the necessary new learning, including a time line. The evaluator may recommend that you take a quantitative reasoning self-assessment prior to developing your plan. The diagnostic will help determine if your wish to complete the competency through independent learning is realistic. It will also provide you and your evaluator with additional information for developing your plan for acquiring the necessary new learning.

When your plan has been finalized and the evaluator has signed off on it, you undertake your study, meeting periodically with the evaluator to go over what you have done and to get feedback. When you complete your study, you will be asked to take a quantitative reasoning evaluation exam, which will have two components:

a. a proctored test; and,
b. a response to an open-ended question that would demonstrate the depth and breadth of your knowledge in relation to the competency. This response may be completed on your own time and it may take one of three formats: a 2-4 page paper; a multimedia presentation with appropriate documentation; or, a portfolio of prior work.

Based on your performance, your quantitative reasoning evaluator will either certify completion of the competency demonstration or will specify additional work that you will need to complete through

independent study.

3. CPCS Instructional Activity: You take a course or workshop that addresses the Quantitative Reasoning competency, and satisfactorily complete the assignments.

TYPES OF PROBLEMS TO DEMONSTRATE THE QR COMPETENCY CRITERIA

In our various evaluations, in addition to covering all the competency criteria, we made sure to cover the basic math literacy skills in various types of questions (i.e., open-ended, reading words and charts and other visual information, problems involving numerous calculations), and a range of public and community service issues. Nyabanyaba's (1999) research with math teachers who used word problems with relevant information found that "The ultimate end for the teacher of bringing students' everyday experiences to bear in mathematics classes is mathematics learning rather than the greater illumination of that reality by mathematics." (p.14) In our CPCS context, the reverse is true. Criticalmathematical literacy involves the ability to ask basic statistical questions in order to deepen one's appreciation of particular issues, and the ability to present data to change people's perceptions of those issues. The main goal of a critical understanding of numerical data is to prompt us to question "taken-for-granted" assumptions about how our society is structured, and to enable us all to act from a more informed position on societal structures and processes.

BACKGROUND QR KNOWLEDGE

According to Karl Menninger (1969), throughout history poets and philosophers have been fascinated by zero, the magical digit which creates so much and yet stands for nothing. He quotes a Hindu saying: "Ten men live in such a way that they allow one to take precedence. Without this one they have as little meaning as zeros, unless they are preceded by a 'one.'" Write briefly about what this saying means

- Literally, that is, in terms of the use of zero in the place-value system of writing numerals;
- Figuratively, that is, how the literal fact about the use of zero in the place-value system relates to people organizing themselves in groups;
- Do you agree or disagree with this saying, and why?

CRITERION 1A

Discuss the main claim made in this article and the supports for that claim, spelling out the details in terms of the reasoning using the concept of percent.

Table 1 – Example: criteria 1A

95% TRUTH-FREE (NUTRITION ACTION, DECEMBER 1990)
Lasagna with Meat Sauce that's 95 percent fat-free. Sirloin Salisbury Steak that's 95 percent fat-free. How does Budget Gourmet slim down these and other classic fat-laden favorites?
It doesn't.
The company simply slapped 'ninety-something percent fat-free' claims on some of its Light Entrees. No fuss, no muss.
Trouble is, the lasagna still gets 39% of its 300 calories from the 13 grams of fat in each serving. The Salisbury steak gets 45% of its 260 calories from its 13 grams of fat. In fact, it's virtually identical to Stouffer's ordinary, no-light claims Salisbury Steak.
When Budget Gourmet says these dishes are 95% fat-free, the company isn't lying. Much of the weight in most foods comes from water, so the percentage of fat by weight is low. Entrees are similar to whole milk, which is 96% fat-free by weight. Half of milk's calories come from fat.
The difference is that the Federal Trade Commission has prohibited the American Dairy Association from calling whole milk 96% fat-free because the claim implies that milk is low in fat, and that's deceptive.

In contrast, the U.S. Department of Agriculture approved Budget Gourmet's claims. Nice to know USDA is on the job, protecting us from tricky labels.

Source: Author´s file

CRITERIA 1A AND 2

Use the information in the following table to draw some conclusions about production and non-supervisory workers employment conditions from 1967 to 1997.

- Explain why the average hourly earnings went down only 8.5% from 1973-1997, while the average weekly earnings dropped 14.2% in that same time period.
- List some other quantitative questions you would want answered in order to get a better picture of these workers employment conditions.

Table 2 – Hourly & weekly earnings of production and nonsupervisory workers, 1967-1997 (1997 $).

YEAR	AVERAGE HOURLY EARNINGS	AVERAGE WEEKLY EARNINGS
1967	11.85	450.28
1973	13.40	494.39
1979	13.36	476.97
1989	12.50	432.63
1992	12.09	415.96
1997	12.26	424.20
% CHANGE 1973-97	-8.5%	-14.2%

Source: Economic Policy Institute, *The State of Working America* [sic] 1998-1999.

CRITERIA 1A AND 1B AND 3:

- What is the topic of "Junkie Arithmetic"?
- Restate, in your own words and without using the specific numbers, the information in this piece.
- Calculate whether the USA or Holland has the higher rate of drug addiction per capita.
- Comment on the tone of the piece, giving specific examples of the language used to illustrate your points.
- Why are the addiction rates expressed as "per 10,000" rather than "per 100 (i.e., percent)"?

Table 3 – Example: criteria 1A, 1B and 3

JUNKIE[7] ARITHMETIC (*EXTRA! Update*, June 1995)

New York Post TV columnist John Podhoretz (4/6/95) attacked *ABC News'* look at *America's War on Drugs* as "emotionally infantile" and "addlebrained," citing the network's look at how other countries handle their drug problems: "Holland claims a very low rate of addiction, but that is nonsense," the conservative critic wrote. "According to the show, there are 25,000 government-registered heroin addicts in a nation of 15 million. Here in the United States there are approximately 500,000 junkies in a nation of 259 million. Guess who has way, way more junkies per capita? Holland does. (I realize the math is hard for the people over at *ABC*.)" Jim Ledbetter pointed out the obvious in the *Village Voice* (4/18/95): These figures yield an addiction rate of 16.6 per 10,000 in Holland and a *higher* rate—19.3 per 10,000—in the U.S. As Podhoretz asked of *ABC*: "What are you, stupid over there?"

Source: Author´s file

CRITERION 1C

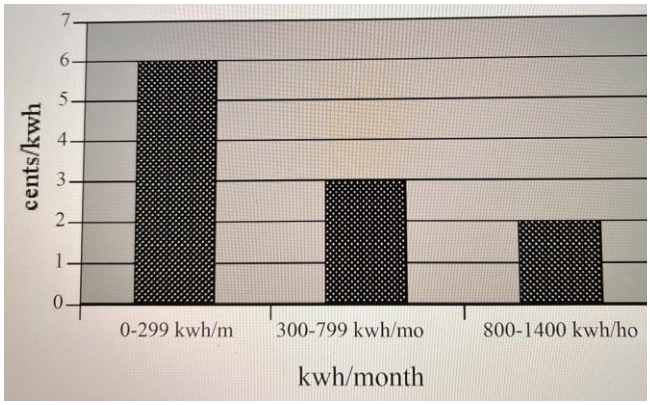
The Rate Watcher's Guide (Morgan, 1980) details why under declining block rate structures, low-income citizens who use electricity only for basic necessities pay the highest rates, and large users with luxuries like trash compactors, heated swimming pools or central air-conditioning pay the lowest rates. A 1972 study conducted in

[7] When I taught this problem, if none of my students commented on the use of the term "junkies," I raised the issue of language. Why do we call people with the health problem of addiction by a term that references things we do not value, things we want to throw away?

Michigan, for example, found that residents of a poor urban area in Detroit paid 66% more per unit of electricity than did wealthy residents of nearby Bloomfield Hills. Researchers concluded that "approximately $10,000,000 every year leave the city of Detroit to support the quantity discounts of suburban residents." (p. 28)

- What calculations would you need to perform to understand how this kind of rate structure transfers money from the poor to the rich.
- What other kinds of payment structures could be instituted bsides declining block rates?
- Which would you support and why?

Figure 1 – Example of a declining block rate structure for electricity[8]



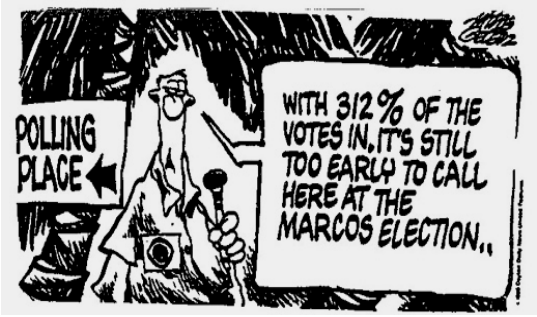
Source: Author´s file

CRITERION 1D

Even if you do not have the background information about this particular electoral situation, try to use your knowledge of percent and of how democratic elections are supposed to work to answer the following questions.

- What is the joke in the political cartoon below[9]?
- What do you need to understand about the concept of percent in order to understand this joke?

Figure 2 – Example citerion 1D



Source: Author´s file

CRITERION 2

In the USA the unemployment rate is defined as: the number of people unemployed divided by the number of people in the labor force. The figures below are from December 1994 (in thousands, rounded to nearest hundred thousand) of various groups of workers. First, consider who you think should be counted as unemployed, who should be considered part of the labor force and why. Then, calculate the unemployment rate based on your decisions.

[8] I used this example in a two part workshop I gave at The Royal Danish School of Educational Studies in 1999. I remarked that in many communities in the USA, environmental and other groups had successfully gotten the government to force companies to change their rate structures, and that possibly this kind of declining block structure was not used anymore in the USA. The next day, one of the workshop participants said he looked closely for the first time at his utility bill and found that it was based on a declining block rate structure! He intended to protest.

[9] Unfortunately I could not read the name of the cartoonist to contact them for permission. If someone knows who did this cartoon please contact me (marilyn.frankenstein@umb.edu) so that I can write for permission.

Table 4 – Criterion 2

NUMBER (THOUSANDS)	CATEGORY
(1) 101,400	Employed full-time
(2) 19,000	Employed part-time, want part -time work
(3) 4,000	Employed part-time, want full-time work
(4) 5,600	Not employed, looked for work in last month, not on temporary layoff
(5) 1,100	Not employed, on temporary layoff
(6) 400	Not employed, want a job now, looked for work in last year, stopped looking because discouraged about prospects of finding work
(7) 1,400	Not employed, want a job now, looked for work in last year, stopped looking for other reasons
(8) 3,800	Not employed, want a job now, have not looked for work in the last year
(9) 60,700	Not employed, don't want a job now (adults)

Source: Authr´s file[10]

THE QUANTITATIVE REASONING SELF-ASSESSMENT

In order to start the process of centering students in their learning, we developed a QR self-assessment where students could first individually decide what was the best way for them to demonstrate the QR competency. They worked on this self-assessment after a few lessons in an introductory course that all students took: Dimensions of Learning. In these lessons they addressed general issues about testing, including:

- What does self-Assessment mean to you?
- General aspects/values of self-assessment, including breaking down the dichotomy between teaching and learning
- Problems in general with testing, the gate-keeping functions of math tests, Gould (1981) on problems with "intelligence" testing, power and knowledge connections
- Writing assignments such as "When I think about math I feel_____"
- Reading assignments—such sharing journal entries on their feelings about math with other students
- Analyses of math 'anxiety' as misconceptions about learning math (Frankenstein, 1989) chapter 1, *Relearning Mathematics: A Different Third R—Radical Maths*.
- Culture of the math textbook (Winter, 1991)
- Political analyses from excerpts such as the study about teacher expectations for black and white students who were labeled "average" and "superior" (Sklar, 1993)
- What studying means, including class discussion of "The Act of Studying" (Freire and Macedo, 1987, p. 76-77), and student practical difficulties with studying, such as time
- Reviewing the QR self-assessment notes to students about learning
- Scheduling a meeting with a QR

[10] Note: Class discussion brings out that there is political struggle involved in deciding who counts as unemployed (and once that is decided, changing the fraction to a decimal to a percent does not involve political struggle, but instead is based on the way the numbers work). In 1994, the USA official definition counted (4) and (5) as unemployed and (1) through (5) as part of the labor force, giving an unemployment rate of 5.1%. If instead we count (4) through (8) plus half of (3) as unemployed, the rate would be 9.3%. Discussions also bring out other groups we could count as partially unemployed, such as the 2.5 million people who worked full-time, year round in 1994 and earned below the official poverty line, (Sklar, 1995).

teacher to review their self-assessment, and negotiate the best method to complete their work on the QR competency

In addition to providing the material references above, the QR teachers discussed various broad goals with the Dimensions of Learning teachers to prepare them for teaching those lessons. In addition to the obvious goal of figuring out how to help students work on their Level 1 and 2 criticalmath literacy competency demonstrations, other goals involved:

- Introducing students to an important and difficult learning/study skill—self-assessment
- Beginning the long process of changing the messages many students (and some faculty) have internalized that they cannot "do math"
- Beginning the long process of changing the (student and faculty) culture about the importance of QR in understanding all areas of study at CPCS
- Beginning the long process of changing the (student and faculty) culture about what math is and what math can do (i.e., challenging myths about how the institutions in our society are structured)
- Beginning the long process of changing the social relations between teachers and students

THE STUDENT SELF-ASSESSMENT:

QUANTITATIVE REASONING [11]

The self-assessment started with a cartoon from the popular comic strip "Peanuts" where Charlie Brown was taking a true or false test, not paying attention to the substance of the problems, but trying to guess a pattern of true and false answers. Charlie concluded that smart people can pass such tests "without being smart," or, really, by using their smarts to figure out the pattern of how the people who made the test would arrange the true and false answers.

We are all "smart." The question is *how* we use our "smarts." We can use them as Charlie Brown does above, to "get over" or we can use them to study—to understand, re-create, analyze, and create—and to act to produce more justice in our world.

Part of the struggle for justice involves assessment, understanding what the real situation is, or, in this case, self-reflection about your basic statistical knowledge. Being 'smart' is not already knowing everything, but rather a process that includes knowing what you already understand, and knowing what questions you need answered in order to understand more deeply.

And part of the struggle to change the world involves quantitative reasoning.

QR SELF-ASSESSMENT—COMMENTS

Remember: The self-assessment that follows gives you an overview of the range of problems you will be able to solve *after* you complete the QR Level 2 competency. The answer section includes suggested solutions and notes to guide your reflections. You are

[11] Note: For purposes of illustration, I have not included all the problems. Further note that there was also a Level 1 Understanding Arguments Self-Assessment, but for the purposes of this article, I focused on the more quantitative Level 2 competency.

not expected to be able to solve these problems *NOW*. You are expected to try each problem and identify, as precisely as you can, everything about it you understand and where you get stuck. And you are expected to think about what parts of the problem are hard or easy for you, and other aspects of your study process. The answers contain suggested solutions and explanatory notes to guide your self-reflections.

The purpose of doing this now, is to help you decide how to prepare for work on the QR Level 2 competency, as well as to prepare for QR work throughout the CPCS curriculum. The last item in the packet is a summary self-reflection and self-recommendation.

The important, and difficult, ideas and skills to begin working on now, and to continue working on throughout your experience at CPCS, are:

- You are not "stupid" if you do not know how to solve a particular problem
- There is always some correct reasoning in any thoughtful solution to a problem. Maybe you know some numerical fact is missing, but you don't know how to find it. Maybe you know the answer needs to be bigger than the given numbers, but are not sure how to calculate it.
- There is always something you can understand about a particular problem, even if most of it is confusing to you. Maybe you can restate the problem in your own words. Maybe you know that the solution involves calculating a percent, but you are not sure which numbers to use.
- To express exactly what you do understand about a problem.
- To learn a language to express precisely the questions you need answered in order to solve the problem.
- There can be many equally correct ways to solve a given problem
- To develop the resources (teachers, tutors, other students, friends, texts, and so on) to help you with your studies.
- To begin to understand the importance of quantitative reasoning in understanding public and community service issues

Note: Some of the questions are open-ended; others involve asking you to create the questions from given numerical information. This is because when you are using quantitative reasoning in order to better understand public and community service issues, in essence, what you need to do is:

- To find the available statistical information
- Understand what it means
- Know what other information would need to be gathered in order to understand the situation more completely
- Know what other questions, including non-numerical ones, need to be answered about the situation in order to have enough information from which to act

Final Note: Self-assessment also involves knowing how you are reacting to what you are doing. To some extent, all learning is stressful. New knowledge shakes up all your prior knowledge. Eventually, you learn how to balance the challenges of new learning and the learning actually becomes fun! One way of helping that process is to reflect on your reactions—is working on a particular problem hard or easy? Why? Is the problem

interesting or boring? Why? How can you make it more interesting? How can you make it more easy? The summary worksheet in the self-assessment packet includes questions like these for you to consider.

QR SELF-ASSESSMENT—THE QUESTIONS AND SUGGESTED SOLUTIONS

For each problem, write your draft notes, and the steps of your solution, not just the answer, in the spaces provided. (Of course, attach additional sheets if needed.) Then, after you work on each problem, check the suggested solution, so that you will remember your reasoning when you worked on that problem. By checking as you work, you will be reinforced in your knowledge, and you will be able to learn from your mistakes. After you review the model solution, add (in another color pen) comments, corrections, questions to your original solution. It is not cheating to change your answers after you gain new knowledge! We ask you to let us know what you added after checking the suggested solution so that we can better advise you about your further study plans. Finally, answer the self-reflection questions about how well you now understand the problem and your experience in working on the solution.

You are not expected to be able to solve these problems until you complete the QR Level 2 competency. And also, remember that for each of your incorrect answers, there is always some correct reasoning. Try to identify at least one thing that you did correctly, as well as your specific mistakes. If your solution and/or answer is different from the suggested solution answer, it does not necessarily mean that you are wrong. It is a misconception that there is only one way to solve and one way to answer every real-life quantitative problem. On the contrary, your method can be different and still take into account all the factors needed for a correct solution; your answer will depend on what assumptions you make related to the given information—some problems are open to a number of different interpretations. So if you have a different method or answer from the one suggested in the given solutions, and after carefully considering those solutions, and reviewing your own solution, you still feel you are correct, discuss it with someone else until you determine whether or not both answers are correct. Do not assume that you are wrong, until you understand your solution and any errors in reasoning or calculations you may have made.

Problem 1: In order to dramatize the toll of domestic violence, on 2/14/95, a bell was sounded in the State House's Great Hall every 15 seconds (*Boston Globe*). Each ring represented that somewhere in the US during that time interval, another woman had been beaten.

- At this rate, calculate the number of women victims of domestic violence during one year.
- Do you think this is an effective way to present these statistics? Why or why not?

Your notes:

Your solution (before and after studying the suggested solution—write any changes or questions you make after studying the suggested solution, in a different color)

Your reflections—What did you learn from working on this problem?

Suggested solutions—problem 1:

Following are two solutions, both of which use multiplication and division, but in different, equally correct ways. The first is the solution that most people avoid because they

are unclear about the power of division. However, the second solution requires that you realize there are four 15 second intervals in one minute, also a division problem: 60 sec ÷ 15 sec = 4.

First solution: Since each 15 second time interval represents one woman beaten, finding how many 15 second time intervals are in 1 year will tell you how many women are beaten in 1 year. This is a division problem—one meaning of division is to find out how many of the same size groupings (in this case 15 second time intervals) fit into another grouping (in this case 1 year). You divide the grouping which is being broken down (here the 1 year) by the size of the other groupings (the 15 second time intervals) to find out how many of the 15 second groupings fit into the 1 year:

1 year ÷ 15 seconds

In order to complete the division, you need to have the same units of measure, so one way to do this is to find out how many seconds are in 1 year:

365 days/yr x 24 hrs/day x 60 min/hr x 60 sec/min = 31,536,000 sec/yr
(Note: One meaning of multiplication is to find a total when you have many groupings of the same size—for example, to find the total number of hours in a year, we use the fact that each of the (approximately) 365 days in the year can also be viewed as a grouping of 24 hours.)

So, now we can divide:

1 yr ÷ 15 sec = 31,536,000 sec ÷ 15 sec = 2,102,400 women beaten in 1 year

(Remember: We calculated the number of 15 second time intervals in 1 year, and because of the equivalency stated in the problem, we can conclude that this translates into 2,102,400 women beaten in 1 year.)

Second solution: You could start by noticing that since 1 min = 60 sec, there are 60 sec/min ÷ 15 sec/woman beaten = 4 women beaten every minute. Then you could use multiplication in order to complete the solution:

4 women beaten/min x 60 min/hr x 24 hr/day x 365 days/yr = 2,102,400 women beaten per year

Comments:

- Notice that both methods involve both division and multiplication. Which method do you think is easier? Why?
- Note that there are other much more cumbersome solutions (like in the first solution, converting everything to years). For a more challenging calculation exercise, think about why the calculations in this other option are so difficult. Then, discuss this with others, including one of the QR teachers.
- Notice that you do not need to know how long the bell was ringing in order to solve the problem—the key thing is the ratio given that for each ring (which equals each 15 sec time interval) one woman is beaten.
- The 15 second figure represents an average—if we assume it is a mean average, calculated by taking the total number of women beaten in 1 year in the USA and dividing by the number of 15 second time intervals in 1 year. Why do you think the people who designed this dramatization decided not to break down the ratio into the average number of women beaten every second? Or every minute? But instead, to break down the yearly figure in 15 second time intervals?
- What other ideas do you have about the way the data was described in the dramatization? What questions do you have about the way the data was collected?

- This problem may have been difficult for you to work on because of the emotional aspects of the facts involved. Some of us believe that being aware of, and critically analyzing the distressing facts about our world is an important step in the struggle to change those distressing facts—knowledge is power, and the first step in gaining the power to change. But this does not mean it is easy to work with such sad data. On the contrary, it is a very hard thing to be able to work with such emotionally difficult data. Some people feel that quantifying a social problem distances us from the emotional issues. What do you feel about the issues of quantifying social problems, of "knowledge is power" and of the awareness of distressing data?

Problem 2: In your own words, creating a different numerical illustration than used in the article, discuss the main claim, the reasons and supporting numerical evidence, for the argument in 'The Gilded Mean.'

Table 6 –Probrem 2

THE GILDED MEAN (IN THESE TIMES, JANUARY 13-19, 1982)

Grasping for a shred of evidence to support their faith, fans of Reaganomics have pointed to the relatively mild decline in average hourly earnings during the present recession. In the year ending last Nov. 1, reports "In Short" financial correspondent George Lowrey, inflation-adjusted earnings did fall "only" 1.5 percent (making the cumulative drop 8.2 percent since 1977).

But there's a madness to the method of computing this figure, which can end up understating the squeeze on workers. Statisticians at the Bureau of Labor Statistics (BLS) first survey the wages of a random group of employees at a selection of companies. Then they divide the total payroll of those workers by hours worked. Such computations may give an accurate reading during the best of times, but a hitch develops during downturns. When a company lays off workers, it usually starts with the lowest paid. These people then drop out of the BLS sample, and the average wage of those who remain is higher than before.

Or consider this example: For September, the BLS surveys three workers, earning $3, $5 and $7 an hour. Average wage: $5 an hour. The fact-gatherers return in October, after the lowest paid employee has been laid off and the other two forced to accept reductions of $1 an hour each. BLS finds two workers making $4 and $6 an hour and calculates that the average wage is still $5 an hour. Eureka! Cries the Reaganomist. Average hourly wages have remained stable, and—despite reports of layoffs and pay concessions—the average worker has not been hurt much by the president's policies.

Source: Author´s file

Your notes:

Your solution (before and after studying the suggested solution—write any changes or questions you make after studying the suggested solution, in a different color)

Your reflections—What did you learn from working on this problem?

Suggested solution—problem 2:

Main Claim: The overarching main claim is that a mean average can be misleading in forming conclusions about workers' wages.

Comment: Another, more specific, claim made in the article is that the belief of Reagan supporters that workers have not been hurt 'too much' (only a 1.5% drop in inflation-adjusted earnings) is not necessarily true.

Reasons Supporting the Claim: The reasons for the general and for the more specific claim are that a mean average can stay almost the same under very different conditions.

Comments:

- Unstated in the article, the mean will remain virtually unchanged, if all the workers' wages remain virtually unchanged.
- The quantitative example in the article shows that the mean can also remain almost the same if the lower paid workers are laid off—even if the remaining workers take pay cuts.

Numerical evidence Supporting the Reasoning: A similar example to the one in the article, where the lowest paid worker is laid off and the other workers get pay cuts: Three workers are paid $4, $7, and $10 per hour. The lowest paid is laid off and the other two are forced to accept reductions of $1 an hour each. The mean average was initially $4 + $7 + $10 = $21 ÷ 3 = $7 and the new mean average is $6 + $9 = $15 ÷ 2 = $7.50! So,

conditions are worse for all three workers and, in this case, the (mean) average hourly wage has increased.

Comments:

- Try to 'play 'with the numbers and come up with some other examples that explore how the mean average can be misleading in other ways.
- The median average (the number in the middle—in this case, half the hourly wages are greater than and half the hourly wages are less than the median; if there are an even number of figures, the mean average is calculated for the two figures in the middle) can also be misleading. Try to 'play 'with the numbers to decide see how that can be the case.

Problem 3: Discuss the main point of the 'We are not a Minority' billboard.

- Do you agree or disagree. Why?
- What quantitative reasoning is involved in this billboard?
- What kinds of numerical data could you use to support or refute the argument made in the billboard?
-

Figure 3 – Los Angeles growing number of Hispanics and other recent immigrants are demanding their piece of the pie.

Source: Author´s file

Your notes:

Your solution (before and after studying the suggested solution—write any changes or questions you make after studying the suggested solution, in a different color)

Your reflections—What did you learn from working on this problem?

Suggested solution—problem 3:

Main Point: Of course, a picture may be "worth a thousand words," but it is certainly open to a thousand interpretations! In this case, the most literal interpretation is that Latinx in Los Angeles are not a minority, quantitatively, of the entire population. A more qualitative analysis of the word "minority" would deepen this interpretation—Latinx are not "minority" in their perspectives, or humanity. A more political interpretation, using the Che Guevera iconic portrait, is that Latinx are a powerful force to be reckoned with—not a "minor" group to ignore.

Comments:

- Many aspects of understanding the point of this billboard involve prior knowledge—that Latinx make up the vast majority of the population in areas of L.A. and that the picture is a well-recognized representation of Ché Guevera, a well-known Argentinian revolutionary (fighting alongside Fidel Castro in the Cuban Revolution, for example). You are ***not*** "stupid" if you do not know this information—rather, you just need to ask, or do some other kind of research, to learn whatever background information you need.
- When I presented this as an example at a talk I gave at CPCS, one of the faculty felt that this billboard was very aggressive. One of my former students pointed out that if he were a stranger to the neighborhood, he would feel the billboard was threatening; however, if he were a resident of the neighborhood, he would feel the billboard symbolized the pride of the community, the non-marginalization of the members of that Latinx neighborhood in L.A.
- Recently, the Boston Globe (9/16/2000) carried a story about

Alberto Diaz Gutierrez (more well-known as Alberto Korda), the photographer of the image of Ché used in the above billboard. He had just won copyright protection, in a case against advertisers who had used the Ché image in an advertisement for Smirnoff vodka. Although Gutierrez has never made any money from his photograph (one of the world's most widely reproduced images), he brought the lawsuit so that the Ché icon will only be used to promote the cause of social justice—"the ideals," Gutierrez said, "for which Ché Guevera died." Gutierrez, who lives in Havana, will donate the settlement and any other proceeds from the Ché photograph to children's medical care in Cuba. Do you think this billboard, using the image of Ché, is effective as political art? Is it a powerful visual image? Why or why not?

Quantitative Reasoning: Often numbers are behind economic, political, or social issues even if there are no numbers "visible" in the picture. In this case, if we knew the population statistics for Latinx and other groups, we could calculate the percent of the entire population that each group represents. It is necessary to understand that percents start as fractions, as comparisons between two groups. Then the fraction can be rewritten as a decimal and then as a percent. Also, the percents of Latinx will differ according to the base—Los Angeles, the United States, the world. In 1990, in East L.A., for example, there were 119,684 Latinx out of a total population of 126,379. So, Latinx were 119,684 ÷ 126,379 ≈ 0.947 = 94.7% of the population of East L.A.—certainly not a minority! In the state of California in the same year, Latinx were 7,687,938 ÷ 29,760,021 ≈ 25.8% of the entire population. Was this a "minority"?

Comments:

- It is not possible to determine if Latinx were a "minority" in California without quantitative information about the percent of the entire population made up by each of the parallel demographic categories (i.e., what percent of the population were Asian-American? European-American? African-American? Indigenous American?)
- An interesting thought about the politics of language world-wide is that Latinx and other people of color are often referred to as "minorities" when in 1986 the population of Africa and Asia alone, mostly people of color, made up 70 percent of the world's population. Some people currently use the phrase "people of the global majority" instead of "people of color." I prefer the former phrase since it is informational, not just descriptive.

Summary self-reflections and self-recommendations for further study

Self-reflections: Review your entire QR Self-Assessment packet. Write briefly about what you learned from the entire process. Also comment on your experience studying the problems—what was the hardest, what was the easiest problem, and why? What was the most interesting, what was the least interesting problem, and why? What suggestions do you have for us to improve this QR Self-Assessment packet?

Possible Self-Recommendations to discuss with a QR teacher:

- I felt confused while solving these problems, even after reviewing the suggested solutions. I will take at least one Level 1 QR-Intensive class (such as Understanding Arguments) and work in the Math Lab at least once a week in order to prepare for work on the required QR Levels 1 and 2 competencies, as well as to prepare for QR work throughout the CPCS curriculum.
- I understood most of the problems *after* studying the solutions. To reinforce that knowledge I will either

take a Level 1 QR-Intensive class or work in the Math Lab once a week before demonstrating the QR Levels 1 and 2 competencies.

- I feel very comfortable with these kinds of quantitative reasoning problems. I will go to the Math Lab to get study packets so I can prepare to demonstrate the QR Levels 1 and 2 competencies through prior learning.

RESPECTING LEARNER'S KNOWLEDGE :BREAKIN G THE DICHOTOMY BETWEEN LEARNING AND TEACHING

Self-assessment is part of a process of creating more respectful relationships between learners and teachers, and between learners and the materials they are studying. Throughout the educational experience there are many other ways to break apart the dichotomy between learning and teaching that deepen this practice[12].

One way is to ask students to reflect on research that usually would be part of math teacher preparation curricula. An example: I ask students to discuss what conclusions they can draw from the results of the following study (Sklar, 1993) and what other information they would want in order to clarify or deepen their analysis of the data:

> 66 student teachers were told to teach a math concept to four pupils—two white and two black. All of the pupils were of equal, average intelligence. The student teachers were told that in each set of four, one white and one black student was intellectually gifted, the others were labeled as average. The student teachers were monitored through a one-way mirror to see how they reinforced their students' efforts. The "superior" white pupils received two positive reinforcements for every negative one. The "average" white students received one positive reinforcement for every negative reinforcement. The "average" black students received one positive reinforcement for every 1.5 negative reinforcements, while the "superior" black students received one positive response for every 3.5 negative ones.

The quantitative reasoning that emerges from thinking about this question involves learning how to construct a chart that clearly presents this data; and, learning about how ratios summarize all the raw data collected. The chart draws attention to the different treatment based on race and supposed intelligence. We then speculate about reasons for the student-teachers' interactions with their pupils and discuss the role of white supremacy in both the different treatment of black and white students and the more negative treatment of the "gifted" blacks than the "average" blacks. We discuss the role of teacher expectations suggested by the different treatment of the "gifted" and "average" whites. We also question the assumption that, for whatever reason, any student should ever get negative reinforcements.

Students ask many questions about the study, such as how did it measure positive and negative reinforcements and were the student-teachers black or white. We also discuss the ethics of these kinds of studies. I cannot answer many of their questions, since my source is not the original research, so students learn that teachers don't know everything, and we figure out together how we might find the answers to the questions they posed. I also encourage students to consider their prior schooling, reflect on similar or different experiences they have

[12] For more examples, and see Frankenstein (1997).

had, and review how those experiences might have formed the negative judgments that they have internalized about their mathematics abilities.

Another way to center students in their own learning, while also seriously evaluating their understandings, is to give short, frequent in-class tests taken directly from homework problems. In this way, students are encouraged to work on the assignments together *really* studying – there is no reason to "copy" someone's homework, but there is incentive to get others to teach you how to do the problems, so you will be able to solve them on the quiz. Additionally, I often had students create and solve their own quiz questions, making sure that they cover the key aspects of the topics we studied reflected in the homework assignment. In class, then, we discuss how to create good questions whose answers illustrate your knowledge. We would evaluate my homework questions, as well as various student created questions, thinking about whether they are hard or easy and why; interesting or boring and why; relevant or irrelevant to the topic studied and why. A goal is to have students thinking about what knowledge and skills the questions are asking them illustrate, rather than just looking for the correct answers. This way, students are involved in evaluation, not just subject to it. (Frankenstein, 1986b)

Underlying Paulo Freire's educational philosophy is a deep respect for peoples' knowledge. To me, that means we recognize the importance of intellectual diversity and challenge mainstream ideas about who *are* the intellectuals. To me, that means we discuss with our students the meaning of Freire and Macedo's (1987) contention that "the intellectual activity of those without power is always labeled non-intellectual" (p.122). And, it means we empower learners to think deeply about what they understand already, what they don't yet understand, and what is the best way to go about understanding more. When we respect the contributions of everyone involved in the learning environment, and we shift some balance of the power dynamics in traditional educational institutions though centering student's responsibility in their learning process, we start building social relationships that support a transition to more justice. The institutional philosophy of CPCS included a vision of the activist role that education can play in creating a just, humane world, and a vision that intellectual and political activism dialectically support each other. The practice outlined in this article is one place to start.

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