CEMeR
Caminhos da Educação Matemática em Revista
2023 • Ano X • V 13 • n. 1 • p. 47-60
ISSN 1983-7399

ARTIGO ORIGINAL/ ORIGINAL ARTICLE

Modelling in Mathematics Education: different ways to practice and understand

Modelagem na Educação Matemática: diferentes modos de praticar e compreender

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ABSTRACT
This paper presents an essay aiming to introduce modelling in mathematics education to teachers who do not know it and summarize some of its characteristics to the ones who already know it. Starting with four practical examples, whose configuration varies from typical textbook tasks to more open and broader projects, the objective in this paper is to illustrate different ways of putting modelling into practice in educational contexts. My argument is that there is relative flexibility in the ways of conducting modelling activities, which gives teachers greater freedom to choose how to practice and to adapt them, according to their conceptions and the conditions of the school or other context in which they work. After the examples, I present some classifications of mathematical modelling activities proposed in the literature, according to the complexity of the task, the way the task is presented, the role played by students and the teacher in the activity, and the central aim in different modelling perspectives. Finally, I present some paths to find the community who works with modelling in mathematics education, both in Brazil and internationally.

Keywords: Modelling in mathematics education, practice, examples.

RESUMO
Apresento, neste artigo, um ensaio que visa introduzir a modelagem na educação matemática para professores que não a conhecem ou fazer uma síntese de algumas de suas características para aqueles que já a conhecem. Começando com quatro exemplos práticos, cuja configuração varia de tarefas típicas de livros didáticos a projetos mais amplos e abertos, o objetivo, no artigo, é ilustrar diferentes formas de colocar a modelagem em prática em contextos educacionais. Meu argumento é que há uma relativa flexibilidade nas formas de realizar atividades de modelagem, o que dá ao professor maior liberdade para escolher como praticá-la e adaptá-la, de acordo com suas concepções e com as condições da escola ou outro contexto no qual ele/ela trabalha. Depois dos exemplos, eu apresento algumas classificações de atividades de modelagem propostas pela literatura, de acordo com a complexidade da tarefa, com a forma como a tarefa é apresentada, com os papeis desempenhados por estudantes e pelo professor na atividade e com o principal objetivo em diferentes perspectivas de modelagem. Para finalizar, apresento alguns meios de encontrar a comunidade de modelagem na educação matemática, tanto a brasileira quanto a internacionaL.

Palavras-chave: Modelagem na Educação Matemática, práticas, exemplos.

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Recebido em 21/03/2022
Aprovado em 31/08/2022

INTRODUCTION

For some decades, mathematical modelling has been a recognized and consolidated trend in Brazilian mathematics education (Araújo, 2010); however, especially among mathematics teachers, it is still common to hear questions such as: What is mathematical modelling in mathematics education? How to organize modelling activities? Could the activity I carried out in my classroom be called a modelling activity? Nonetheless, it is not appropriate to publish one more paper answering such questions directly, since they have been exhaustively discussed in mathematical modelling literature, as shown throughout this article.

In a broader sense, *mathematical modelling* can be understood as the use of mathematical techniques or tools to solve real-world problems. Sometimes called *extra-mathematical world*, "*real-world* is often used to describe the world outside mathematics" (Niss, Blum & Galbraith, 2007, p. 3). In other words, real-world problems are problems from reality — whatever *reality* means — which encompass other issues, concerns, situations, different from what is commonly called mathematics.

Some of the mathematical objects used to solve real-world problems are called *models*, which are mathematical expressions used to describe some process or situation specific to the problem (Niss, Blum & Galbraith, 2007). Examples of models are difference equations, differential equations, functions or even a computational program algorithm (Meyer, Caldeira & Malheiros, 2011). When such activities are conducted in educational contexts, they are called *modelling in mathematics education* and acquire specific characteristics, adapting to different and novel conditions.

Starting with four practical examples, whose configurations vary from typical textbook tasks to more open and broader projects, the objective in this paper is to illustrate different ways of putting modelling into practice in educational contexts. According to Araújo and Lima (2021), using examples is an interesting strategy to introduce modelling or guide students, or teachers, on how to conduct such activities.

Through the examples, I will seek to describe which understandings of modelling are mobilized in each of them, based on answers already given to the previous questions in both, Brazilian (Almeida & Vertuan, 2011; Barbosa, 2004; Meyer, Caldeira & Malheiros, 2011) and international literature (Niss, Blum & Galbraith, 2007; Villa-Ochoa, Castrillón-Yepes & Sánchez-Cardona, 2017). My argument is that there is relative flexibility in the ways of conducting modelling activities, which gives the teacher greater freedom to choose how to practice and to adapt them, according to their conceptions and the conditions of the school or other context in which they work.

Due to that flexibility, there is a huge number of activities that are labelled as modelling, and it becomes necessary to organize and establish relationships between the different forms of practicing modelling within the field of mathematics education. Thus, after the examples, I present some classifications of mathematical modelling activities, according to task complexity (Antonius *et al.*, 2007), the way the task is presented (Villa-Ochoa, Castrillón-Yepes & Sánchez-Cardona, 2017), the role played by students and teachers in the activity (Barbosa, 2004), and the central aim in different modelling perspectives (Kaiser & Sriraman, 2006).

Finally, I present some paths to find the community who works with modelling in mathematics education, both in Brazil and internationally.

Let us start with an example closer to the day-to-day dealings of mathematics classrooms.

EXAMPLE 1: Half-life and radioactive decay

This is a typical textbook example of differential and integral calculus which are present in the exact sciences curriculum of university courses. I chose an exercise proposed by Stewart (2009, p. 222):

Example 1: “The half-life of cesium-137 is 30 $years$. Supposing a sample of 100 mg, calculate the remaining mass after t $years$."

In this example, the real-world problem to be solved is described precisely by the statement. The statement suggests that the mathematical model for solving the problem should be a function: mass m (in mg) of cesium-137 remaining as a function of time t (in $years$). In the context of higher education, the mathematical tool used to obtain that function is a differential equation — which is a type of mathematical model. This exercise is proposed by Stewart (2009) right after presenting the corresponding content. However, in this article we will solve the problem using high school tools and techniques. Some of them are also used at elementary-school level.

The half-life of a substance is the time it takes for half of any amount of the substance to disintegrate. As the statement in the example indicates, the half-life of cesium-137 is 30 $years$. Then, the initial mass of cesium-137 (100 mg) is reduced to half every 30 $years$, following the pattern:

$$m(0) = 100$$

$$m(30) = \frac{1}{2}(100)$$

$$m(60) = \frac{1}{2} \cdot \frac{1}{2}(100) = \left(\frac{1}{2}\right)^2 (100)$$

$$m(90) = \frac{1}{2} \cdot \left(\frac{1}{2}\right)^2 (100) = \left(\frac{1}{2}\right)^3 (100)$$

$$m(120) = \frac{1}{2} \cdot \left(\frac{1}{2}\right)^3 (100) = \left(\frac{1}{2}\right)^4 (100)$$

Thus, successively, from this pattern we can deduce that the remaining cesium-137 mass, after t $years$, is

$$m(t) = \left(\frac{1}{2}\right)^{\frac{t}{30}} \cdot 100 = 100 \cdot \left[\left(\frac{1}{2}\right)^{\frac{1}{30}}\right]^t$$

Therefore the model is an exponential function with base: $= \left(\frac{1}{2}\right)^{\frac{1}{30}} = \frac{1}{\sqrt[30]{2}}$.

Mathematical modelling tasks like this one are classified by Villa-Ochoa, Castrillón-Yepes and Sánchez-Cardona (2017) as problems with *verbal statements,* or *word problems*, as Niss, Blum and Galbraith refer to them (2007). In this type of task, a situation is described via text, followed by a question that may be answered with the support of mathematics. The situation, in turn, may be real or fictitious, and may not necessarily relate to the daily life of the students attempting to solve the problem.

Thus, the example shows that mathematical modelling can be present in textbooks and quite familiar in the daily practice of mathematics classes. However, Villa-Ochoa, Castrillón-Yepes and Sánchez-Cardona (2017) have warned us that such tasks are questioned by the literature regarding modelling in mathematics education. Many authors do not even recognize them as characteristic of modelling. Such questioning takes place because in those tasks the real-world problem is usually fictitious and only "disguises mathematical content under a garment of words." (p. 232). According to the authors, such characteristics are different from the authentic problems that originate in real-life situations, which are quite different from exercises intended to illustrate the application of mathematical content.

In the next example, let us deal with a real-world problem more in tune with the literature regarding modelling in mathematics education.

EXAMPLE 2: Finding out the shoe size

This example is described by Tortola and Silva (2021). It was conducted in an 4th-grade elementary school class. After performing an arts task, involving the *Abaporu*[1] painting by Tarsila do Amaral, the children wondered about the shoe size of the person depicted in the painting, as proportionally the feet were much larger than the head. This is how the real-world problem was set: What is the relationship between a person's foot size and their shoe size?

To solve the problem, the children started by collecting data: they drew their own feet on sheets of paper, measured the lengths of their outlined feet, and tried to set relationships with the sizes of their shoes. After some unsuccessful attempts, the teacher told a story of how Europeans established shoe sizes as the number of barley grains that fit, lined up, in the length of a foot.[2] Although the European shoe numbering system is different from the Brazilian system, the story brought new impetus to the students' investigations. They decided to run tests using beans and rice grains. By means of practical experiments, questions about grain length, verification of the need to determine standard units of measure, and discussions regarding the calculations required to mathematically express that relationship, the class concluded that the shoe size is obtained by dividing the length of the foot by the average length of a grain of rice, which after the experiments, was adopted by the class as $0.67\ cm$.

The activity reported in this example was performed by fourth-graders who, therefore, did not know how to generalize with the support of algebraic tools. Thus, in the context of the first years of elementary school, the mathematical model (Tortola & Silva, 2021) obtained by the class was the sequence of measurements and operations required to solve the real-world problem.

This is an example of the adaptations mathematical modelling undergoes when it is resignified in mathematical education. Mathematical models are now understood in a broader way, as described by Almeida and Virtuan (2011, p. 21):

> a conceptual, descriptive, or explanatory system, expressed through a language or a mathematical structure, with the purpose of describing the behavior of another system and allowing predictions about it.

This very same problem could be solved in the final years of elementary school with the support of the concept of function: N, the shoe size, can be described as a function of C (in cm), the foot length, by the expression

$$N = \frac{C}{0.67}$$

In this case, the model would be a linear function.

Unlike example 1, example 2 is linked with the students' daily life, although it is still a relatively short activity that can be carried out in one single mathematics class, and which requires simple mathematical content.

[1] *Abaporu* is a classic of Brazilian modernism; considered the masterpiece of the artist Tarsila do Amaral. The oil on canvas painted in 1928 was gifted to her then husband, the writer Oswald de Andrade.

[2] Tortola and Silva (2021) explain the whole process with details and illustrations. I invite readers to read the article.

As Meyer, Caldeira and Malheiros (2011, p. 29) have warned us, the notion "that a highly sophisticated, elaborated mathematical context is an indispensable condition for modelling" is not true.

Antonius *et al.* (2007) would classify this example as an *investigation*, in which, at first, students do not know how to solve the real-world problem; so, they set out to investigate.

For Tortola and Silva (2021), the activity they describe belongs to the *educational perspective* (Kaiser & Sriraman, 2006) of modelling in mathematics education, whose aim is to teach mathematical content through modelling activities.

Another aspect worth highlighting in this example is the possibility of involving different disciplines — arts, mathematics, and history — in the same activity. Mathematical modelling has a strong potential to foster interdisciplinarity (Silva *et al.*, 2021).

Teaching mathematical content might not always be the goal when a modelling activity is performed, as illustrated by the next example.

EXAMPLE 3: Simulations of the value of minimum wage

I was involved in the conduction of the modelling activity described in this example, as well as in Araújo and Martins (2017). The context is much more complex than that of the previous examples: the year was 2016, right after the impeachment of President Dilma Roussef. Vice-president Michel Temer, who was sworn in her place, forwarded to Congress a series of proposals for changes in Brazilian law. One of such changes was the *Proposta de Emenda à Constituição* (Proposed Amendment to the Constitution) number 55, or PEC55, which imposed limits on public spending, determining that, each year, the maximum rate for adjusting government spending would be the official inflation index.

As a form of protest, during the proceedings for passing PEC55, students from all over the country occupied schools and universities, which also occurred in the building of the *Instituto de Ciências Exatas* (Institute of Exact Sciences) (ICEx) of UFMG, where I teach. The *Grupo de Discussões sobre Modelagem na Educação Matemática* (Discussion Group on Modelling in Mathematics Education) (GDMEM) which is based at ICEx, offered a modelling workshop to complete the framework of activities carried out during the occupation of ICEx. That is the origin of the name Workshop *#OcupaICEx.*

GDMEM held a first meeting with representatives of the occupation movement, aiming to submit our proposal and select a real-life theme to be addressed in the workshop. Considering the reason for the occupation, the representatives chose the theme of PEC55 to be addressed in the modelling activity. When the theme was chosen, GDMEM began to prepare for the workshop: they studied PEC55, sought reports and testimonials, conducted investigations about it, made simulations, and, finally, delimited a real-world problem, related to the theme: What would the value of minimum wage be at that time — November 2016 — if it were calculated as per the standards set forth by PEC55?

At this point, I wish to make two remarks, comparing this example with the previous ones. The first remark is about the number of lines I used up to the point of presenting the real-world problem in each of the examples. There were five lines in example 1, nine lines in example 2, and over forty lines in example 3. In this case, the more detailed presentation was necessary to describe the social and political richness of the context in which the modelling activity was conducted.

The second remark was the choice of a broader theme, PEC55, before defining a real-world problem. Although the previous examples were not preceded by the choice of a theme, this situation is common, particularly in mathematical modelling *projects* (Antonius *et al.*, 2007; Villa-Ochoa, Castrillón-Yepes & Sánchez-Cardona, 2017). The choice of a theme is foreseen, for example, by Burak (2019), who proposes five stages to perform modelling activities: 1) choosing the theme; 2) exploratory research; 3) defining the problem(s); 4) solving the problem(s) and developing mathematical contents related to the theme; 5) critically analyzing solutions.

Back to our story, the participants of the workshop *#OcupaICEx* were mainly students of mathematics, physics, and chemistry at UFMG, somehow involved with the occupation of ICEx. GDMEM members organized the workshop: we distributed tables with minimum wage values and the *Índice Nacional de Preços ao Consumidor Amplo* (National Broad Consumer Price Index) (IPCA); organized the participants into groups; guided and directed the work, clarifying doubts and raising mathematical, social, and political questions. At the end of approximately one hour, some groups presented their solutions, all different from each other.[3]

Different mathematical strategies could be used: elementary operations with the data contained in the tables, performed with pencil on paper, or simple calculators; calculations with the support of spreadsheets, etc. For example, a mathematical model that could be used, recursively, is:

$$S(n+1) = \left(1 + \frac{i_n}{100}\right).S(n)$$

In this model, $S(n)$ is the value of the simulated minimum wage, in the year n, and i_n is the sum of the IPCA index values of July of year $n-1$ to June of year n, supplied by one of the tables. This model was very accessible to workshop participants, who were higher education students of exact sciences. However, it is possible to obtain solutions and models as appropriate as that one with the support of more elementary mathematical content.

Similarly to example 2, example 3 relates directly to students' daily lives, but has a few more ingredients. In this case, the goal was not to teach mathematics to workshop participants, as they had already fully mastered the content. Nonetheless, as much as the participants were proficient with the mathematical content, they did not feel safe discussing the situation based on mathematical arguments, because "social and political discussions are not usually included in mathematics classes". (Araújo & Martins, 2017, p. 125). The goal of GDMEM was to discuss social and political issues, raising critical aspects, reflecting, and problematizing such situations with the support of mathematics. It is, therefore, an activity from a *socio-critical perspective* of modelling in mathematics education (Araújo, 2009; Barbosa, 2006; Kaiser & Sriraman, 2006). This is also why I presented more details of the social and political context in which example 3 was inserted.

As they have a similar logistics organization, though based on issues of a different nature, Salazar *et al.* (2017) propose a new reading of the stages of Burak for the organization of modelling activities within

[3] A possible solution to the problem is described by Araújo and Martins (2017). I invite readers to check the article.

the *socio-critical perspective*: 1) choosing a theme or problem, giving significant importance to macro and micro contexts; 2) developing exploratory research; 3) collecting data and designing routes of action; 4) reinterpreting the situation, with the support of mathematical considerations, and developing the problem; 5) critically analyzing the proposed developments.

This example is also different from the others because the students participated more in the definition of the modelling activity. In example 1, the problem was a task from a textbook which did not rely on the participation of students; in example 2, the teacher proposed the problem, although she encouraged students to get involved with it; in example 3, the theme was proposed by the students in the occupation and GDMEM — a group of teachers — delimited the problem.

This greater responsibility on the part of students could bring the task closer to what Barbosa (2004) calls *case 3* of mathematical modelling, in which a theme from reality is chosen by the students. However, *case 3* requires students to participate in the formulation of the problem, data collection and problem resolution. This was not the case in example 3, as the teachers provided the problem and data. *Case 3* is best illustrated by the following example.

EXAMPLE 4: Line at the restaurant at UFMG

One of the tasks I usually propose in disciplines dedicated to modelling in mathematics education is as follows: I ask the students, gathered in groups, to choose the real-life theme or problem to be addressed in a modelling activity. As a teacher, I try to understand the choices made by the groups and, starting from such choices, I guide them in attempting to achieve the desired goal. This type of task illustrates *case 3* as proposed by Barbosa (2004).

Once, a group of mathematics undergraduates proposed the following real-world problem: Why does the restaurant at UFMG always have available seats, at any given time, even though the number of users is greater than the total number of seats in the hall?

The group decided to go to the restaurant and collect data to help them find an answer to the problem. One day, at lunchtime, they went to the restaurant to i) gauge the time a person stood in line, until being served, ii) count the number of seats available, iii) count the number of users entering the restaurant every minute, and iv) calculate the average time it took for a person to have lunch. They collected some other data, but the above were the main ones.

I do not have the record of these data as I returned the report to the group after evaluating it. However, I remember that the group understood what was happening and was satisfied with the answer they found to the problem with the support of the data. Using simple counting, with data collected in a single day, they understood that the time spent standing in line, which also included the time waiting to be served, did not allow many people to enter the restaurant at the same time. This time was also longer than the average time a person spent eating. As these times offset one another, they prevented all seats from being taken at once.

The group was pleased with the calculations and answer; however, I think that they could have advanced more in terms of the generality of the solution. Some of the data obtained by the group remained constant, such as the number of seats in the restaurant, but others varied as a function of time. However, the group performed measurements at small time intervals. For example, if a person arrived at the restaurant at 11:00 a.m., the time they would stand in line might be much shorter than that of the person arriving at 12:00 or 12:30. Therefore, if we denote the time a person stands in line as T (in min), and as h the time the person joins the line, we could attempt to write T as a function of h, i.e., $T = f(h)$.

Similarly, the number of people entering the restaurant every minute also varies depending on the time of day. In this case, we would have the rate of change, $N'(t) = \frac{dN}{dt}$, of the number of people, N, who enter the restaurant per minute, as function of time t. We must also consider the number of people who leave the restaurant per minute and the fact that T is related to t.

What would the solution to this real-world problem be if the group decided to take this other route to mathematical modelling? As a professor and researcher, I suspect that we would have had more significant learning opportunities.

The first challenge we would have faced would have been to demystify the concept of function, as we would not have an algebraic expression for $T = f(h)$ nor for $N'(t) = \frac{dN}{dt}$, nor for other functions involved in the problem. Would we try to approximate discrete data through a continuous function, using some software, or would we work with the original functions, with discrete data? How would we calculate the integral of $N'(t)$, which is not expressed by means of a formula? Surely, the students would increase their knowledge of mathematics from dealing with these questions conceptually, as they demand much more than the application of formulas.

Another challenge would have been mathematical modelling itself. The group presented a particular solution to the problem they proposed, since they collected data for only one day. The solution I am proposing would be more general and would require collecting data throughout a longer period, as well as an analysis of which variables the problem would involve.

Depending on the emphasis that the group and I, as the teacher, gave to each of these queries, we could classify the modelling activity as pertaining to the above-described *educational perspective*, or to the *realistic perspective* (Kaiser & Sriraman, 2006), in which the goal is to understand and solve a real problem effectively, in such a way that students improve their problem-solving skills in the real-world. We could also raise some critical questions about whether it would be reasonable to spend so much time standing in line instead of studying. In this case, the activity could take on a *socio-critical perspective*.

In addition to that, we must not forget that the real-life problem to be solved "is a fundamental part of the mathematical modelling process" (Dalla Vecchia & Maltempi, 2019, p. 750) and it is determinant to the richness of the discussions and conduction of a modelling activity. It is important to recognize the merit of the group who proposed the restaurant problem with potential to trigger significant learning. I invite readers to solve this problem, adapting it to the reality of their university or school.

After presenting the examples, I will address them collectively in the following section.

The examples

The four examples presented in the article were chosen to illustrate different ways of practicing and understanding modelling in the field of mathematics education. Example 1 was taken from a higher education textbook and required no student participation; example 2 was conducted with 4th grade students; example 3 took place within a protest movement led by university students; and example 4 was proposed by a group of mathematics undergraduates. Each example belonged to a different context and displayed unique characteristics, which depended on the context, the teacher, and students, but all of them originated from a real-world problem that was solved with the support of mathematics.

The complexity of the mathematical content mobilized to perform modelling in each of the examples was also different. Purposely, I described or suggested different paths to solve each of the real-world problems, signaling the flexibility of the approach for each problem, which can be solved using elementary or more elaborate mathematical content, but always in dialogue with the actual situation or the theme in which the problem was inserted. The fact that contents belonging to other disciplines (other than mathematics) were treated in mathematics classes did not prevent mathematical content from being treated in classes. The reader must have realized, however, that the mathematical content used in the examples is not limited to application of formulas. It is necessary to know these contents conceptually to enable the bidirectional exchange between the situation from which the problem derives and mathematics.

In theoretical terms, I tried to discuss each of the examples with the support of different studies, bringing elements of literature regarding modelling in mathematics education that could describe and enable the understanding of the situations reported. As it should have become clear throughout the examples, there is not just one way to analyze modelling practices or classify them. In the next section, I present some of these forms.

Some classifications of modelling activities

Antonius *et al.* (2007) organized the variety and complexity of application and modelling activities into three types, based mainly on the length of the activity: tasks in mathematics and applications; investigations; projects.

Tasks in mathematics and applications are shorter and can be carried out in one single class. Some of them are similar to tasks presented in curriculum proposals or mere illustrations of applications in mathematics. *Investigations*, in turn, are a little longer, lasting a few classes and demanding problem-solving skills from students. Finally, *projects* can extend for weeks and are much more complex activities, “usually addressing a broader problem embedded in the real-world” (Antonius *et al.*, 2007, p. 297).

Based on practices reported in the literature, particularly from the statement of the tasks that trigger the practices, Villa-Ochoa, Castrillón-Yepes and Sánchez-Cardona (2017) proposed another categorization of modelling tasks: verbally stated problems; construction of representations; modelling through projects; use and analysis of models. Some of these categories have subdivisions.

Verbally stated problems — or *word problems* — are tasks in which a non-mathematical situation is described in a text, and a question to be answered mathematically is posed. The problems are usually familiar to students. The authors subdivide this category into two: realistic verbal statements and authentic problems presented as verbal statements.

In modelling tasks, there is usually a need to create a mathematical representation for variables or relationships present in the real-world situation. When emphasis is placed at that moment, Villa-Ochoa, Castrillón-Yepes Sánchez-Cardona (2017) classify the task as *construction of representations*. They explain that this can happen depending on the purpose of the task or the way the teacher understands mathematics teaching. This category is subdivided into graphical representations and simulations of forms.

Modelling through projects occurs in broader activities, in which open problems, with non-predictable routes, are proposed to students. Villa-Ochoa, Castrillón-Yepes e Sánchez-Cardona (2017) describe different purposes in the development of projects: teaching certain content, developing students' capabilities, establishing relationships between mathematics and other disciplines, or carrying out reflections on the role of mathematics in society.

Finally, the authors approach the *use and analysis of models*, which presupposes the consideration of models already established, to analyze what is at stake: variables, parameters, how they relate and describe mathematically the real-world situation; rather than constructing a model.

Looking at the people involved in modelling activities, Barbosa (2004) describes three cases, according to the roles played and responsibilities assumed by students and teachers. To this end, the author chooses four steps that are usually included in modelling activities — problem formulation, simplification, data collection, solution — and the cases are described according to the responsibilities assumed in each of such steps.

In *case 1*, the teacher is responsible for formulating the problem, simplifying, and collecting data. All this information is provided to students who work solely in the search for solutions to the problem. In *case 2*, the teacher formulates and proposes the problem to the students. The other steps are performed by the students, with constant guidance of the teacher. Finally, in *case 3*, students are responsible for all four steps. They are the ones who formulate the problem, simplify it, collect data, and seek solutions. The teacher, in turn, acts as an advisor in all four steps.

Based on the literature regarding modelling in mathematics education, Kaiser and Sriraman (2006) present a modelling activity classification system, according to the central aim in different modelling perspectives. The authors propose five perspectives: realistic, contextual, educational, socio-critical, and epistemological; and one meta-perspective: cognitive.

The *realistic* and *contextual perspectives* are oriented to the non-mathematical situation involved in the modelling activity. The *realistic perspective* has pragmatic and utilitarian objectives, aiming to create skills, in students, to effectively solve real-world problems. The *contextual perspective*, in short, aims to answer that old prevalent question in mathematics classes: where will I use this content? That is, the purpose is to show applications of mathematics, in areas outside it, through problem solving.

The *educational* and *epistemological perspectives* are more oriented to mathematics, with mathematical modelling activities as a reference. The *educational perspective* aims to teach mathematical content using modelling, promoting learning processes, or introducing new concepts. The *epistemological perspective* aims to promote the mathematical theorization involved in the modelling activity. Its purpose is also to develop theories.

The *socio-critical perspective* is oriented to the role of mathematics in society, seeking to enable students' critical reading of the world through mathematics, questioning absolutist conceptions of the discipline.

Lastly, the *cognitive meta-perspective* is called that "because it is not a normative approach connected to goals of teaching modelling in school" (Kaiser & Sriraman, 2006, p. 307), taking a more descriptive role of how modelling processes take place.

As the title of this section indicates, these are some possibilities for classifying modelling activities. Certainly, there are others. By presenting these possibilities, my intention is to show that, depending on the researcher's view, different modelling activities nuances, and purposes can be highlighted. I do not see as advantageous to take any of these classifications in a deterministic way, particularly regarding teacher practice. The classifications show possibilities, and they can be the inspiration for teaching practice.

The classifications have been illustrated in the descriptions of the examples and I summarize them as follows: example 1 could be classified as a *word problem* (Villa-Ochoa, Castrillón-Yepes & Sánchez-Cardona, 2017); example 2 could be classified as an *investigation* (Antonius *et al.*, 2007), belonging to the *educational perspective* (Kaiser & Sriraman, 2006); example 3 could be classified as a modelling *project* (Antonius *et al.*, 2007), in the *socio-critical perspective* (Kaiser & Sriraman, 2006), and typical of *case 1* (Barbosa, 2004); example 4 would illustrate *case 3* (Barbosa, 2004), and could belong to the *educational perspective*, to the *realistic* or to the *socio-critical perspective* (Kaiser & Sriraman, 2006), depending on the emphasis given by students and the teacher.

As we near the end of this article, the readers must have realized the vast and diverse literature available on mathematical modelling, both in print and electronic form; on websites, dissertations and theses repositories, or proceedings of congresses. But where can one find the people who do this research and author these papers? The next section contains some suggestions.

Where to find the modelling in mathematics education community?

The authors of the works in which I based the reflections presented herein are part of the modelling in mathematics education community. Both the Brazilian and international communities are always open to dialogue with mathematics teachers, future teachers, and researchers who are interested in our pedagogical practices and research. When one intends to start pedagogical practices or research involving this theme — or any other — it is important to know what has already been produced about it in the literature.

Most of such authors or researchers, maybe all of them, are members of research groups, which are nuclear organizations dedicated to discussion and research. Internet searches allow us to contact these research groups. For example, the *Conselho Nacional de Desenvolvimento Científico e Tecnológico* (National Council for Scientific and Technological Development) (CNPq) maintains a directory of Research Groups in Brazil, whose data are available on the internet.

In Brazil, the modelling community is organized around the *Grupo de Trabalho de*

Modelagem [4] (Modelling Working Group) (GT10) of *Sociedade Brasileira de Educação Matemática* (the Brazilian Society of Mathematics Education) (SBEM). The website of SBEM states that

> the mission of GT10 is to foster debate and collaboration of Brazilian researchers who conduct investigations on mathematical modelling from the perspective of mathematics education, contributing to the development of this research field in the country.

One of the responsibilities of GT10 is to organize the biannual meetings of *Conferência Nacional sobre Modelagem na Educação Matemática* [5] (the National Conference on Modelling in Mathematics Education) (CNMEM). Since 1999, CNMEM congregates teachers, researchers, and students interested in modelling, to present papers, participate in thematic debates, or short courses. The GT10 is also responsible for activities at *Encontro Nacional de Educação Matemática* (the National Meeting on Mathematics Education) (ENEM), and *Seminário Internacional de Pesquisa em Educação Matemática* (the International Seminar on Research in Mathematics Education) (SIPEM). The website of SBEM has information on these events.

Worldwide, the modelling community is organized in the International Community of Teachers of Mathematical Modelling and Applications [6] (ICTMA), an organization affiliated to the International Commission on Mathematical Instruction (ICMI). ICTMA holds its own biennial conference. Information about it is available at its website.

Among other initiatives, GT10 and ICTMA also organize thematic issues of journals, and books on modelling in mathematics education. Barbosa, Caldeira and Araújo (2007), and Almeida, Araújo and Bisognin (2011), for example, are books organized by GT10. The proceedings of the ICTMA conferences are published in the series of books International Perspectives on the Teaching and Learning of Mathematical Modelling [7], Springer Publishing Company, organized by Gabriele Kaiser and Gloria Stillman.

Lastly, Table 1 lists thematic issues regarding modelling in mathematics education published in scientific journals.

By means of the examples, the classifications of modelling activities and the introduction to the modelling in mathematics education community, I invite the reader to join us in the practice and research of this field of mathematics education.

Table 1 – Thematic issues of scientific journals on modelling in mathematics education

Journal	**Thematic issue**
ZDM – The International Journal on Mathematics Education	v. 38, ns. 2, 3, 2006
Alexandria: Revista de Educação em Ciência e Tecnologia	v. 2, n. 2, 2009
Bolema – Boletim de Educação Matemática	v. 26, n. 43, 2012
Acta Scientiae – Revista de Ensino de Ciências e Matemática	v. 14, n. 2, 2012
Revista de Matemática, Ensino e Cultura – REMATEC	v. 9, n. 17, 2014
Revista Eletrônica de Educação	v. 19, 2014

[4] GT10 site within SBEM website: http://www.sbembrasil.org.br/sbembrasil/index.php/grupo-de-trabalho/gt/gt-10
[5] XI CNMEM website: http://eventos.sbem.com.br/index.php/cnmem/2019
[6] ICTMA website: https://www.ictma.net/
[7] Information available at: https://www.springer.com/series/10093

Matemática – REVEMAT	
Educação Matemática em Revista	n. 46, 2015
Educere et Educare – Revista de Educação	v. 12, n. 24, 2017
Revista Latino-americana de Etnomatemática	v. 11, n. 1, 2018
ZDM–The International Journal on Mathematics Education	v. 50, ns. 1, 2, 2018
Revista de Educação Matemática (SBEM-SP)	v. 16, n. 21, 2019
Revista de Ensino de Ciências e Matemática (REnCiMa)	v. 12, n. 2, 2021
Quadrante – Revista de Investigação em Educação Matemática	v. 30, ns. 1, 2, 2021
Revista Paranaense de Educação Matemática	v. 10, n. 23, 2021
Educational Studies in Mathematics	v. 109, n. 2, 2022

Source: Author.

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