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ARTIGO ORIGINAL/ ORIGINAL ARTICLE

Mapping research about the multiplicative structures field based on the conceptual field theory

Mapeamento de pesquisas sobre o campo das estruturas multiplicativas na perspectiva da teoria dos campos conceituais

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ABSTRACT
This article is an excerpt from a completed master's research and aims to present a mapping of research on the Field of Multiplicative Structures from the perspective of the Theory of Conceptual Fields. To this end, we consider a corpus of master's and doctoral research defended within the period (1997-2018) available in the Database of Dissertations and Theses of CAPES. We used the following search descriptors: "multiplicative structures" and "Multiplicative Field" and obtained 97 studies. After an initial analysis, we observed that only 64 studies met our established parameters. Considering this corpus, we organized the data to show the concentration by type of study, year, and publication data. Subsequently, we categorized the studies considering the participants or focus of the studies. The mapping showed few studies had been developed on combinatorial reasoning from Vergnaud's theoretical perspective, many barriers to learning combinatorial concepts, and students have great difficulties understanding those concepts.

Keywords: Conceptual Fields Theory. Combinatorial Reasoning. Mapping.

RESUMO
O presente artigo é um recorte de uma pesquisa concluída de mestrado e tem por objetivo apresentar um mapeamento de pesquisas sobre o Campo das Estruturas Multiplicativas na perspectiva da Teoria dos Campos Conceituais. Para tanto, consideramos como córpus de análise pesquisas disponibilizadas no Banco de Dissertações e Teses da CAPES, considerando as pesquisas de Mestrado e Doutorado defendidas dentro do período (1997-2018) e os descritores de busca, "estruturas multiplicativas" e "Campo Multiplicativo", resultando em 97 pesquisas. Após uma análise inicial, observamos que apenas 64 pesquisas atendiam os parâmetros estabelecidos. Considerando esse córpus, organizamos os dados buscando evidenciar a concentração por tipo de estudo, ano e lócus de produção. Posteriormente, categorizamos os estudos considerando os participantes ou foco dos estudos. O mapeamento evidenciou que ainda foram desenvolvidos poucos estudos sobre o Raciocínio Combinatório na perspectiva teórica de Vergnaud e, ainda, que há muitas barreiras na aprendizagem dos conceitos combinatórios e que os alunos apresentam grandes dificuldades na compreensão desses conceitos.

Palavras-chave: Teoria do Campos Conceituais. Raciocínio Combinatório. Mapeamento.

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INTRODUCTION

According to Vergnaud (1990), Conceptual Field is a set of situations whose treatment implies closely connected schemes, concepts and theorems – the ability to solve problems in the most diverse situations where one finds a given concept is not developed within months, not even within years.

Accordingly, Borba (2013) states that closely related concepts in teaching can be simultaneously approached, since situations that give meaning to these concepts are entangled; moreover, it is possible progressively working the most complex aspects of the aimed contents.

Based on Borba (2013, p. 2):

> different situations that give meaning to Combinatorial – such as Cartesian product, arrangement, combination and permutation problems – are closely associated through basic Combinatorial relations that must be treated through symbolic representations that allow the adequate screening of possibilities.

Although problems related to Combinatorial belong to the Conceptual Field of Multiplicative Structures, they are not common multiplicative problems, they are more complex and, overall, they are not solved through direct multiplication. It is clear that Combinatorial is a subject addressed in high school, when it is approached based on an excessive and unnecessary amount of formulas; therefore, it is very important to have an approach in elementary school about both the Fundamental Counting Principle and combinatorial problems of the following types: arrangement, combination, permutation and Cartesian product.

Pessoa and Borba (2009) carried out a study with Basic Education students and observed the performance of students in the second and third grades of elementary school who were asked to solve two combinatorial problems of each type (arrangement, combination, permutation and Cartesian product). Its results showed that children in the aforementioned grades get to perceive the features of combinatorial problems. However, students in the second grade of elementary school had a hard time exhausting all the possibilities. Students in the third grade, in their turn, could not get to the end of the solutions, even when results were higher than 20.

Thus, given the likely challenges and difficulties inherent to the teaching and learning process of some mathematical contents, which are faced by both students and teachers, such as the case of Combinatorial Reasoning, we opted for mapping (Fiorentini, Passos, & Lima, 2016) the Brazilian studies that have investigated the Multiplicative Structures Field based on the Conceptual Fields Theory by Gerard Vergnaud. Such a mapping was carried out based on research available in Capes' Bank of Dissertations and Theses.

The present mapping may not have included all research carried out in Brazil about the Multiplicative Structures Field, since this study type never ends, once we are dependent on information that authors have declared as research focus: keywords, titles and, most of all, abstracts. Titles, alone, did not help the clear understanding about the work in some research, and it forced the present researchers to read all abstracts.

Next, we introduce the criteria and procedures triggering the mapping composition.

CRITERIA AND PROCEDURES FOR MAPPING COMPOSITION

We opted to explore Capes' Bank of Dissertations and These to substantiate the present study. We took into account the Master's and PhD research defended between 1997 and 2018, as well as the following search meshes: "multiplicative structures" (which led to 70 studies) and "Multiplicative Field" (which resulted in 27 research). Accordingly, and by taking into consideration the search criteria, the total of 97 studies were initially found: 66 were Professional Master's dissertations and 17 were PhD theses.

After defining this *corpus,* we built a database with the dissertations and theses files - we considered some essential elements to build the referred mapping, such as ID; Course (Master's or PhD); main focus; title; author; year; program; institution and abstract. Thus, we created an electronic spreadsheet in Excel software to organize the collected data.

After we had this set of organization instruments, we started reading the 97 abstracts. It is important pointing out that the elements we looked for were not always clear in the abstracts. Thus, we had to read the whole research in order to structure the herein proposed mapping.

After reading all titles/abstracts and/or research, we observed that only 66 studies addressed the Multiplicative Structures Field within the Mathematics teaching and/or learning context. Besides, it is essential highlighting that two studies were not fully available. Given the hard time finding them, we decided for not including them in the present mapping. Therefore, we started analyzing a set with 74 studies.

After reading the abstracts and texts of the selected studies, we summarized them to identify some investigation aspects. We elaborated an Analysis Matrix to analyze the abstracts based on the following elements: study identification code; course; title; researcher; adviser; Post-graduation Program; field; Institution; state; year; research problem; aim; theoretical references; research focus (student/teacher/materials); data collection and analysis procedures; main results.

Next, we introduce a brief analysis about the studies that have investigated the Multiplicative Structures Field based on the Conceptual Fields Theory by Vergnaud. We will detail general aspects and production *locus*, results deriving from data search and collection processes, and collected-data organization.

A SIGHT OVER THE MAPPED RESEARCH

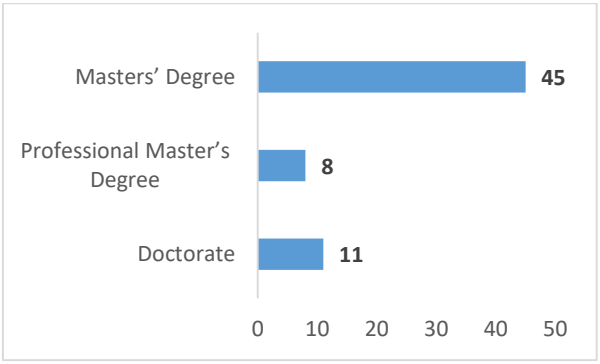
The aim of this topic is to introduce general aspects and production *locus*, such as number of research per level, number of academic productions per year - from 1996 to 2019 -, University and Brazilian region where the studies were carried out. Finally, we will highlight some findings and/or results, and the researchers' investigation focus; in other words, research subjects.

a) General aspects and production *locus*

Graphic 1 presents the 64 studies found during our search; they were separated into modalities of Master's Degree course (Professional or Academic) and PhD. Thus, we started identifying the perspectives and trends presented by these Brazilian studies, between 1997 and 2019, about the

Multiplicative Structures Field based on the Conceptual Fields Theory by Vergnaud.

Graphic 1 – Research that have investigated Multiplicative Structures based on the Conceptual Fields Theory by Vergnaud



Source: Elaborated by the authors (2020).

We have observed that the number of Master's dissertations (Professional or Academic) represent more than 80% of the defended research. We can see a much smaller number of PhD theses. This finding is likely related to the fact that most Post-graduation programs that have given birth to a larger number of Master's research do not offer Doctorate courses.

Next, chart 1 depicts the number of academic productions per year – of the total of selected research, 54 were Master's Degree research (Professional and Academic) and 11 were PhD research.

Chart 1 – Distribution of research concerning Multiplicative Structures, based on the Conceptual Fields Theory by Vergnaud, from 1997 to 2018.

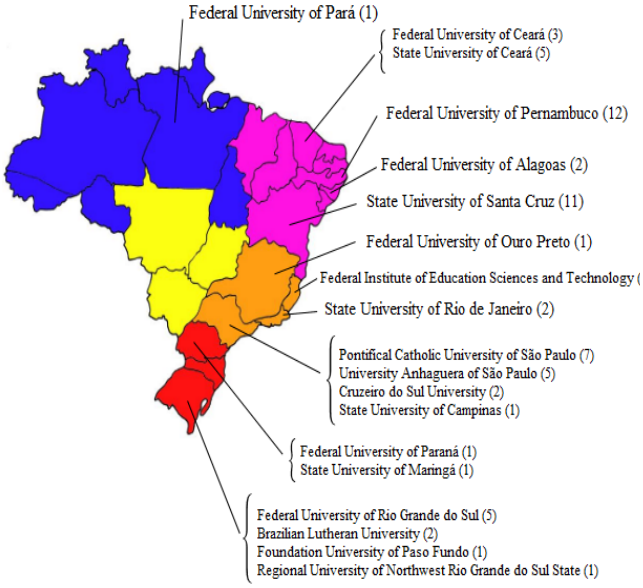
Year	Doctorate	Professional Master's Degree	Masters' Degree
1997	0	0	1
1998	0	0	0
1999	0	0	1
2000	0	0	0
2001	0	0	0
2002	0	0	1
2003	0	0	0
2004	1	0	0
2005	0	0	0
2006	0	0	2
2007	0	1	0
2008	1	0	2
2009	1	1	2
2010	1	1	3
2011	0	0	0
2012	0	1	2
2013	0	0	4
2014	0	1	1
2015	0	0	6
2016	4	2	5
2017	1	1	9
2018	2	0	6

Source: Elaborated by the authors (2020).

Based on the analyzed period, Graphic 2 shows that, for the first 10 years, only six studies were carried out, whereas this production significantly increased in the second decade: 39 studies. It is the outcome of the growing interest in this subject and of the increased amount of Post-Graduation programs in Education, Mathematics' Education, Mathematics' Teaching and Sciences and Mathematics' teaching. We can also observe that 2017 was the year accounting for the largest number of defended Master's Degree research.

Data presented in the current mapping evidence that the topic about Conceptual Field Concepts and about the Multiplicative Field propositions by Vergnaud has been standing out over the last two decades, mainly in the last four years. Figure 1 depicts the country regions and Universities where these studies were most often carried out.

Figure 1 – Number of carried out research (per University) and per Brazilian region



Source: Elaborated by the authors (2020).

Accordingly, Figure 1 shows 33 studies in Northeastern Brazil; we observed some evidences that may have generated this quite significant number for this region. Universities' adhesion to the OBEDUC [1] project led to the creation of a project named "Study about the number of Multiplicative Structures in Elementary School (E-Mult)", which was developed in partnership with Bahia, Ceará and Pernambuco states. This project resulted in a large academic production. Researchers (Rute Elizabete de Souza Rosa Borba, from State University of Santa Cruz; Sandra Maria Pinto Magina, Vera Lucia Merlini and Eurivalda Ribeiro dos Santos Santana), who act in this field and in Mathematics' Teaching programs of Federal University of Pernambuco, were the ones most cited as study advisers.

The other studies were carried out in the Northern, Southeastern and Southern regions, they accounted for 1, 19 and 11 research, respectively. It is worth highlighting that there was no research carried out in Midwestern Brazil.

Still about Figure 1, it was also possible observing the states and universities where these studies were carried out. The largest concentration of these studies was at Federal University of Pernambuco/PE: 12 studies. This institution was followed by State University of Santa Cruz/BA, with 11 studies and by Pontifical Catholic University/SP, with 7 studies. This number represents 46.87% of all the academic production in this field; in other words, almost half of studies come from only 3 of the 19 universities that were included in this mapping. Thus, Pernambuco and Bahia states are the most representative ones, and they are followed by São Paulo State.

Public institutions accounted for 73.43% of all research, and it is explained by the fact that they concentrate more than 80% of all post-graduation programs in Brazil – for each five Post-graduation programs in the country, four are in public institutions [2]. Private institutions, in their turn, represent 26.56% of the academic production.

Subsequently, we will introduce the research subjects and/or objects. It will not be possible addressing them all because studies in this mapping are not the object of our study, but understating likely gaps and trends in the Multiplicative Structures Field based on the Conceptual Fields Theory in order to better conduct our Master's Degree dissertation.

b) Research subjects and/or Investigation object

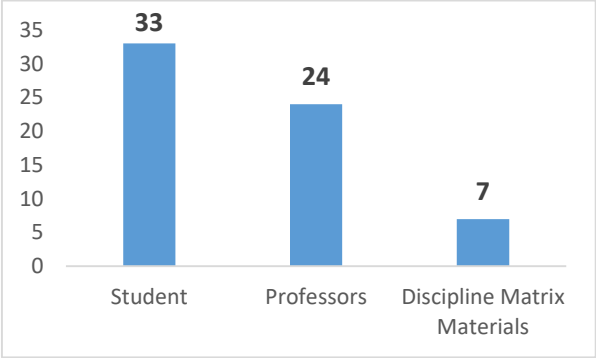
After organizing the physical aspects,

[1] The Education Observatory Program (OBEDUC) resulted from the partnership among Capes, INEP and SECADI. It was launched in 2006 to foment studies and research in Education. OBEDUC aims at allowing articulation among post-graduation, bachelor courses and basic education schools to encourage academic production and training

[2] Source: Sucupira Platform, Capes

such as number of studies, production year, institution and country region, we started a brief content analysis to assess what were the research subjects and/or investigation objects to read the abstracts, or even the full text of the selected research. Graphic 3 depicts such a distribution.

Graphic 3 – Research segmentation per research subject and/or investigation object



Source: Elaborated by the authors (2020).

After performing the necessary readings, we realized that we had students as research subject. Besides, all analyzed research adopted the qualitative research approach with explanatory, interventionist or case study features.

It is worth highlighting that we observed 24 studies about Mathematics' teachers as research subjects. Overall, we observed that these studies aimed at investigating teachers' knowledge, beliefs or teachers who have participated in teacher training programs, based on Vergnaud's propositions about conceptual fields, mainly about Multiplicative Structures.

Finally, 7 studies had the different uses of discipline matrix materials as investigation object. Accordingly, these studies focused on the analysis of didactic books, discipline matrix propositions and the discipline matrices from different Education Secretariats, besides teaching handbooks and the National Discipline Matrix Parameters.

Based on the aforementioned, we aimed at describing the schooling level these studies focused on. In order to do so, we elaborated Charts 1, 2 and 3, based on chronological order. Our goal was to highlight the research subject and /or investigation object, their respective teaching levels and their authors. As aforementioned, we aimed at introducing a panorama of the selected research, without the intention to exhaust the discussion about their investigation focuses.

i) Research about students at different schooling levels

Based on Chart 1, we talk about research subject "student" and about the schooling level of the respective research. We observed that the studies used students as main research subject; they focused on the early grades of elementary school – they represented 28.12% of the mapped research; the late grades of it represented 20.31% of the studies, as well as very low rates of Youngsters and Adults' Education (EJA recorded 3.12% and only 1.56% of the studies focused on higher education students).

Chart 1 – Distribution of research whose subjects were students.

Involved subjects	Schooling level	Authors
Student	Early elementary school	Silva (1999); Batista (2002); Guimarães (2004); Placha (2006); Santana (2008); Silva (2010); Backendorf (2010); Lima (2012); Ferreira

		(2012); Zaran (2013); Fiore (2013); Sena (2015); Oliveira (2015); Teixeira (2016); Melo (2017); Nascimento (2017); Santana (2018); Oliveira (2018).
	Late elementary school	Bonanno (2007); Barbosa (2008); Rasi (2009); Fioreza (2010); Porto (2015); Pereira (2015); Ferraz (2016); Castro (2016); Leite (2016); Dias (2016); Almeida (2017); Aguiar (2017); Pires (2018).
	EJA	Lima (2010); Lima (2018).
	Higher Education	Carvalho (2017)

Source: Elaborated by the authors (2020).

Based on the aforementioned, we could conclude that more than 50% of the selected research had students as research subject; studies forming this axis are research with teaching situations, interventions or propositions that assess teaching and learning processes about Multiplicative Conceptual Field. There were 29 Master's Degree dissertations among studies comprising students as research subject, and this number suggests that there is no continuity between Masters and PhD research.

ii) Research involving teachers' practice and training

With respect to research presenting teachers as research subject, Chart 2 shows 23 studies focused on teachers who act in the early and late grades of elementary school. We have noticed lack of studies related to higher education professors; there was only one research where High school teachers were addressed. The rate between dissertations and theses that had professors as research subject reached 35.93%; they investigated professors' training, beliefs and knowledge. The following chart shows the authors and the year their research was concluded.

Chart *2* – Distribution of research whose subjects are the professors.

Involved subjects	**Schooling level**	**Authors**
Professors	Early elementary school	Canôas (1997); Santos (2006); Vasconcelos (2008); Silva (2009); Yamanaka (2009); Alencar (2012); Silva (2014); Borga (2015); Maia (2016); Lima (2016); Castro (2016); Brito (2017); Luna (2017); Santos (2017); Santos (2017); Oliveira (2017); Silva (2018).
	Late elementary school	Souza (2015); Barreto (2016);

		Milagre (2017); Correia (2018).
	Late elementary school and early high school	Moreira (2014)
	High school	Medeiros (2018)

Source: Elaborated by the authors (2020).

However, we have evidenced that studies adopting professors as research subject were outnumbered in comparison to studies related to students as research subject - it was possible finding 5 PhD theses. This finding shows the prevalent interest in this topic and research continuity in this subject.

iii) Research involving Discipline Matrix Materials and Theoretical Studies

Finally, we developed the topic about studies that have assessed Discipline Matrix Materials, such as didactic books; official documentation analysis, mapping and theoretical studies about the Multiplicative Conceptual Field. They represent 10.9% of all mapped research. Subsequently, Chart 3 shows the composition of these studies.

Chart *3*: Distribution of research involving Discipline Matrix Materials.

Investigation Object	**Discipline Matrix Material Type**	**Authors**
Discipline Matrix Materials	Didactic Book	Filho (2009); Martins (2010); Castro (2016).
	Documental analysis	Niemann (2013); Soares (2016).
	Mapping	Beyer (2018).
	Theoretical Study	Zanella (2013).

Source: Elaborated by the authors (2020).

We have concluded that most researchers who adopted students or teachers as research subject assessed the early grades of elementary school. After creating the EBEDUC project, it was possible evidencing that most research resulted from the project called "Study about the domain of Multiplicative Structures in Elementary School (E-Mult)", which is linked to Universities in Northeastern Brazil.

Finally, basic operations were the mathematical context explored in the mapped research, mainly multiplication and divisions, ratio, rate, function, fraction, percentage and combinatorial reasoning. Only four studies presented a more specific approach about the combinatorial reasoning topic. Next, we will describe these studies in details, since they are close to the focus of our investigation.

iv) Studies that have investigated Combinatorial Reasoning based on the Conceptual Fields Theory by Vergnaud.

As aforementioned, the first part of the current research's development lied on mapping dissertations and theses about the Multiplicative Structures Field based on the Conceptual Fields Theory by Gerard Vergnaud. Accordingly, it was possible getting to know the academic production on this topic. The herein carried out mapping allowed us to get to know the performed studies, as well as to better understand the meanings of expressions associated with combinatorial reasoning, their theoretical reference and the ideas that have emerged from research reading.

After the mapping process was consolidated, we observed a small number of

works whose approach regarded the Combinatorial Reasoning topic based on the Conceptual Fields Theory by Vergnaud. Thus, the mapping process enabled proving this gap and, consequently, it helped us emphasizing the relevance of the present study

Furthermore, because such studies are close to our study object, we aimed at describing them by taking into account aspects like investigation question, data production methods and some of our main results. However, first of all, we introduced Chart 4, and the synthesis of all studies that have approached Combinatorial Reasoning based on the Conceptual Fields Theory.

Chart *4*: Studies that have approached the topic: Combinatorial Reasoning

Title	**Author**	**year**	**Institution**	**Course**
The solution of product of measure problems for children in the third grade of Elementary School and intervention by professor.	Kelly Cristine Placha	2006	UFPR-PR	Master's Degree
Combinatorial reasoning of students in youngsters and adults education (EJA): from early schooling to high school	Rita de Cássia Gomes de Lima	2010	UFPE-PE	Master's Degree
Knowledge about Combinatorial analysis applied to Mathematics teachers: diagnostic study	Francis Miller Barbosa Moreira	2014	UESC-BA	Master's Degree
Combinatorial and probabilistic Reasoning in EJA: investigating relations	Ewellen Tenorio de Lima	2018	UFPE-PE	Master's Degree

Source: Elaborated by the authors (2020).

As shown in the previous chart, all selected studies were developed at Master's Degree Level. Next, we will present a brief description of each one of them in order to identify their contributions to, and likely gaps in, investigation at Mathematics Education scope.

The research by Placha (2006) was based on the following main investigation question: How does the learning process applied to multiplicative relations of product of measures for children in the third grade take place under teachers' intervention? It was an exploratory study, of qualitative nature, that has investigated children's learning process, as well as the learning of multiplicative structures, based on the propositions by Vergnaud about conceptual fields.

Eight multiplicative structure problems of the products of measures type were introduced. The research was carried out with five children from the third grade of Elementary School. The studies by Moro and Soares (2006), which are described at combinatorial reasoning levels and sublevels, were taken as basis to analyze data and results' discussion.

The researcher highlights in discussions about the results that,

> Results in this research point out that children's improvements in the learning process applied to multiplicative relations of products of measures, from less advanced levels of solutions to the most advanced solution levels throughout problem solving, are linked to the intervention ways used by the appraiser (PLACHA, 2006, p. 265).

This researcher highlights some results in the study that she took as fundamental for professors and researchers in the Mathematics Education field.

> [...] the need of teachers to encourage children to use strategies characteristic of calculus; the relevance of the study with mathematical concepts based on problem solving; the importance of having the teacher identifying and following-up the learning process of children so that they can carry out significant interventions: the relevance of children's interpretation about notational and verbal solutions used by them when they solve problems; the importance of working with multiplicative structures in Child Education and in the early grades of Elementary School and the need of having teachers learning the mathematical concepts they teach to children (Placha, 2006, p. 270).

Based on this weighing, this researcher also pinpointed that work with combinatorial reasoning must start from the understanding level the child is at. She aimed at starting from children's understanding level about combinatorial reasoning at problem solving.

We have observed that two studies (Lima, 2010; Lima, 2018) had the same adviser: Rute Elizabete de Souza Rosa Borba, from the Mathematical and Technological Education Program of Federal University of Pernambuco. Both studies collected data about EJA students. Lima (2018), besides addressing combinatorial problems, sought to focus on relations set between different Combinatorial and Probability knowledge.

The research by Lima (2010) aimed at understanding individuals in Youngsters and Adults Education (EJA), at five levels, in this teaching modality, about Multiplicative Structures problems, mainly the ones about combinatorial reasoning of different natures (arrangement, combination, permutation and Cartesian product). In total, 150 EJA students, from 5 institutions (1 municipal, 1 state, 1 federal and 1 institution funded by The Social Service of the Commerce - SESC) participated in the study.

The research was substantiated by the Conceptual Fields Theory and by the Multiplicative Structures by Vergnaud. Thus, a test with EJA students was carried out, it comprised 16 multiplicative and combinatorial questions (2 questions to each problem type) elaborated based on the study by Selva and Borba (2008): "Adding teachers' knowledge about multiplicative conceptual development". Combinatorial problems are part of the study by Pessoa (2009), "Who dances with whom: understanding combinatorial reasoning from the age of 7 to 17 years".

Accordingly, the researcher aimed at introducing her analysis and results discussion based on performance, depending on school grade; on multiplicative and combinatorial problem type, depending on school grade; on years of study, since several of these students have significant disparity between their permanence at school; on exerted profession; on response types presented by students in the modules and in professional activities; and finally, on strategy

types used by students based on multiplicative problem type and on professions. We believe that such detailing is justified by the number of students and by the variation in modules attended by the students.

Lima (2010) states that:

> Cartesian product problems in all modules (grades), mainly the direct product, were the ones presenting the highest rate of correct answers. Based on Pessoa (2009), it can happen due to school's influence, because problems involving combinatorial reasoning are often explicitly approached with children in the 3rd and 4th grades of elementary school; the Cartesian product is approached along with other meanings given to multiplicative structures, such as proportionality, rectangular and comparative configuration (Lima, 2010, p. 92).

This researcher makes an inference regarding the main goals of the research when it comes to comparison of performances based on schooling; she states that:

> Whenever they are related to schooling, it was possible observing that, as we advance in schooling, we also observe upgrades in performance regarding understanding the meanings of problems, i.e., it means that depending on schooling upgrades, students would use the concepts-in-action characteristic of each problem type applied to combinatorial concepts (Lima, 2010, p. 126).

Lima (2018) aimed at analyzing contributions the exploration of combinatorial problems could give to probabilistic reasoning, vice-versa, within the EJA context. The research focused on relations set between Combinatorial and Probabilistic knowledge. It regards a quantitative-qualitative profile intervention aimed at getting theoretical substantiation in the Conceptual Fields Theory by Vergnaud and in literature review, which opened room for Youngsters and Adults' Education (EJA). Accordingly, these authors present and reflect about a summary of discipline matrix propositions for EJA, as well as previous studies focused on this teaching modality.

The aforementioned researcher states that some results were confirmed by findings by Lima (2010), such as schooling: "as schooling upgraded, it was possible observing a better understanding of invariants of problems addressed and the use of symbolic representations and strategies best suited to their resolutions", Lima (2018, p. 78, emphasis by the author).

This researcher shone light on the fact that the articulation between Combinatorial and Probability must be thought in a non-dissociated way, given the development of combinatorial and probabilistic reasoning:

> [...] thus, given what was observed based on the conduction of the present exploratory study with EJA students, it was possible advocating that the articulation between combinatorial and probabilistic reasoning can benefit both in this teaching modality. It was possible observing contributions that emerge between Combinatorial and Probabilistic knowledge, which allow the association between them (Lima, 2018, p. 137).

By going on with the studies that have approached Combinatorial Reasoning, based on the Conceptual Fields Theory, Moreira (2014) investigated the performance and

strategies introduced by Mathematics teachers when they work with problems concerning Combinatorial analysis. The Conceptual Fields Theory by Vergnaud (1996) was adopted as theoretical reference, mainly in the Multiplicative Structures field - product axis of measures, where Combinatorial Analysis is inserted in; as well as the ideas by Shulman (1986, 2005) about the importance of teachers' knowledge for their practice and professional development. In total, 18 Mathematics teachers who were starting their Professional Master's activities in Mathematics in the National Network (PROFMAT) of a public university in Southern Bahia State participated in the research.

The researcher presented the results of its study based on three aspects: Teachers' profile, quantitative analysis of teachers' performance and qualitative analysis of responses and strategy types. With respect to teachers' profile, "teachers are concerned with their training, because, although all of them (except one) had a major degree in Mathematics, 11 have already attended a specialization course; nowadays, all of them are attending the Professional Masters in Mathematics" (Moreira, 2014, p. 144).

With respect to the quantitative analysis of teachers' development, this author states: "we observe that teachers show less difficulty in arrangement situations, it was followed by the Cartesian product, permutation and, finally, by the lowest difficulties in combinatory problems" (Moreira, 2014, p. 145). As for the qualitative analysis of response and strategy types, "the most used strategies were the fundamental principle of counting and formulas, and it has shown teachers' preference for the most formal resolution methods" (Moreira, 2014, p. 145).

Besides, this author highlights that the research met the proposed goal throughout research development; some ideas emerged as likelihood of heading towards future studies. According to Moreira (2014),

> By developing this research to investigate teachers' performance at the time to solve problems about Combinatorial Analysis, we found a chart that shows several gaps in these teachers' knowledge about this content. Accordingly, we asked ourselves to which extent these gaps are related to these teachers' training. In other words, how have concepts related to Combinatorial Analysis been approached in Mathematics teachers' training courses? Thus, we suggest a reflection about the training of future and current teachers who teach Mathematics concerning Combinatorial Analysis (Moreira, 2014, p. 149).

However, by concluding this mapping, it is essential highlighting that, after we identified four studies about Combinatorial Reasoning and a quite smaller number of research on this topic, we assume that we will contribute with one more research to help better understanding the learning process involving Combinatorial Reasoning in the Multiplicative Structures Field, based on the Theory by Vergaund and on other theories focused on combinatorial reasoning.

FINAL CONSIDERATIONS

Overall, the herein carried out analysis of mapped studies evidenced several barriers in the process concerning combinatorial concepts' learning and that students have a very hard time understanding these concepts. Pires (2018), for example, states that the Multiplicative Conceptual Field, in ternary

relations, in the axes of product of measures, in the rectangular and combinatorial configuration, from the problem-solving perspective, remains little approached at late elementary school years, since results based on the analysis of strategies used by students, pointed towards the sum of repeated parts or towards successive subtractions.

It was possible mapping 64 studies by following the elected descriptors, but only 4 of them focused on investigating Combinatorial Reasoning. However, we know that there are other studies (Rocha, 2011; Azevedo, 2013; Vega, 2014; Montenegro, 2018; among others) approaching combinatorial reasoning based on Vergnaud; they are inserted in the Multiplicative Structures Field, but they are not addressed in the current study, due to the descriptors used for the search.

Nevertheless, we believe on the relevance of this topic for the Mathematics Education field, and that our mapping can contribute to theoretical and methodological discussions about the Multiplicative Structures Field based on the Conceptual Fields Theory by Vergnaud and on other theories focused on combinatorial reasoning.

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