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ARTIGO ORIGINAL/ ORIGINAL ARTICLE

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El Papel de las Prácticas de Autorregulación en la Modelización Matemática

The Role of Self-Regulation Practices in Mathematical Modelling

RESUMEN

Una de las habilidades fundamentales que relaciona el conocimiento matemático con el contexto cotidiano de los individuos es la modelación matemática, la cual ha ido ganando terreno en los currículos educativos de varios países. Otra habilidad que ha asumido un papel central en la Educación Matemática es la autorregulación, ya que permite planificar, gestionar y controlar el aprendizaje de forma autónoma. En este contexto, resulta interesante plantear la siguiente pregunta: ¿Cuál es el papel de las prácticas de autorregulación en el proceso de modelación matemática? Para responderla, proponemos una articulación entre dos enfoques teóricos: por un lado, la modelización como competencia y proceso matemático y, por otro, la autorregulación como competencia transversal; ambos aplicados al análisis de la actividad matemática en la modelización. Metodológicamente, se trata de un estudio teórico-reflexivo que comienza con el análisis de la resolución de un problema de modelización realizado por expertos y se complementa con la identificación de prácticas que promueven el aprendizaje autorregulado involucrado en las diferentes transiciones de un ciclo de modelización. Finalmente, esta reflexión se aplicó al análisis de la implementación de un taller que combina aprendizaje autorregulado y modelación para profesores de matemáticas de secundaria en servicio. Como resultado, se propone una articulación entre ambos enfoques teóricos para el desarrollo de la actividad matemática en la modelación, resaltando la importancia de incorporar prácticas que promuevan el aprendizaje autorregulado al trabajar este proceso en el aula.

Palabras clave: *Aprendizaje autorregulado, Formación docente, Modelización matemática.*

ABSTRACT

One of the fundamental competencies to relate the individuals' mathematical knowledge to their daily context is mathematical modelling, which has been gaining importance in educational curricula at international level. Self-regulation is another competency that has taken centre stage in Mathematics Education, since it allows autonomous planning, management, and control of learning. In this context, it is interesting for us to pose the question: What is the role played by self-regulation practices in the mathematical modelling process? To answer it, we propose an articulation between two theoretical approaches: on one hand, modelling as a mathematical competency and process; on the other hand, self-regulation as a transversal competency; both applied to the analysis of the mathematical activity in modelling. Methodologically, it is a reflective-on-theory study in which we start from the analysis of the expert solving of a modelling problem, which we complemented with the identification of the intervening practices that promote self-regulated learning in the different transitions of a modelling cycle. Finally, we applied this reflection to the analysis of the implementation of a workshop that combines self-regulated learning and modelling for practising secondary education mathematics teachers. As a result, we propose an articulation between both theoretical approaches for the development of the mathematical activity in modelling, which highlights the importance of incorporating practices that promote self-regulated learning when working with this process in the classroom.

Keywords: Mathematical modelling, Self-regulated learning, Teacher education.

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INTRODUCTION

In the first two decades of the twenty-first century, literature in Mathematics Education has not only been dedicated to addressing the problems inherent in teaching mathematics at different educational levels but has also deepened and refined the quality of its theoretical constructs. A line of research proposed to address this last issue is the Networking of Theories (BIKNER-AHSBAHS; PREDIGER, 2014), which allows establishing a dialogue between two theoretical constructs of different nature. For example, in the case of two general theoretical frameworks, that is, those that focus on the analysis of mathematical activity in general, the work of Borji et al. (2018) stands out, in which the APOS Theory is articulated with the Onto-Semiotic Approach for the analysis of the understanding of the graph of the derivative at the university level. In the case of a specific theoretical framework, that is, one that focuses on a specific type of mathematical activity or process, with a general framework, the work of Rodríguez-Nieto et al. (2022) stands out, in which the Extended Theory of Mathematical Connections is articulated, as a specific framework, with the Onto-Semiotic Approach for understanding the derivative.

In this line of theoretical development, it is interesting to broaden the view beyond the articulations mentioned above, not only considering theoretical references that emerge from Mathematics Education, but also those that, although not originating exclusively in this discipline, do have a close relationship with the learning of mathematics.

Within Mathematics Education, one of the fundamental competencies to relate individuals' mathematical knowledge with their daily context is modelling (KAISER, 2020). This mathematical competency encompasses the modelling process, which brings with it a series of benefits for the learning of this subject (BLUM, 2011), and is considered an indispensable factor for the education of competent individuals to face contemporary needs and demands (MAASS et al., 2022).

Another set of competencies that must be developed for the learning of mathematics are the transversal competencies, including self-regulation. The development of this competency allows conducting a self-regulated learning, in which individuals have a series of tools to better organise, structure, and control their learning, as well as being able to adapt themselves to various contexts. Literature confirms that those students with a high degree of self-regulation have greater academic success (ALTUN; ERDEN, 2013; CUELI et al., 2013). Also, these investigations show that this competency increases student motivation and enhances self-efficacy in learning (LAVASANI et al., 2011).

In this context, given the importance of modelling and self-regulation for teaching and learning mathematics, it is interesting for us to pose the following research question: What is the role played by self-regulation practices in the mathematical modelling process? To address this issue, in this article we propose an articulation between two theoretical approaches: on one hand, modelling as a mathematical competency and process; on the other hand, self-regulation as a transversal competency; both applied to the analysis of the mathematical activity in modelling.

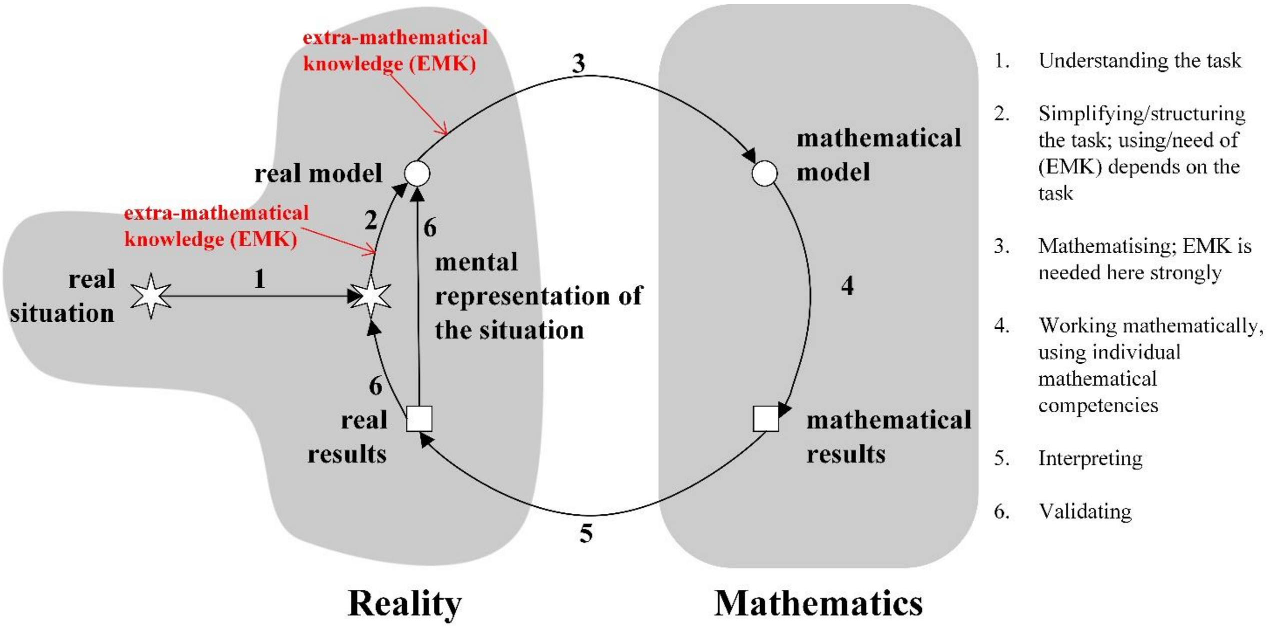
The relevance of this study lies in two aspects. First, we consider two theoretical references of different origins, but that address the problems of Mathematics Education specifically, such as modelling, and transversally, such as self-regulation, broadening the view of the Networking of Theories. Second, we aim to contribute to one of the demands of the modelling research community that recommends deepening the assessment of this competency in students through alternative and diversified methods (see FREJD; VOS, 2024).

Finally, we stress that a proposal like that reported in this article, as far as we know, has not been developed in detail within the specialised literature before.

MATHEMATICAL MODELLING

In general terms, modelling is understood as a process in which a problem-situation is taken from «reality» and solved with the tools of «mathematics», in order to obtain a plausible answer in the context of this problem-situation. In the construction of the theoretical corpus of modelling, different cycles have been proposed to describe this process (BORROMEO FERRI, 2006), as well as different perspectives have been established on its implementation in the classroom (PRECIADO et al., 2023). In this study, we consider the mathematical modelling cycle from a cognitive perspective (MMCCP), proposed by Borromeo Ferri (2018), which is presented in Figure 1.

Figure 1 – Mathematical modelling cycle from a cognitive perspective



Source: Adapted from Borromeo Ferri (2018, p. 15)

The choice of this particular cycle is justified by the authors' experience in its use in previous theoretical articulations (see LEDEZMA; FONT; SALA, 2023), and its operation as a tool to analyse the mathematical activity in modelling is exemplified in the methodological section.

Although modelling is understood as a mathematical process, its development is also considered as a mathematical competency. In this sense, the modelling competency consists of being able to work (build, critically analyse, evaluate) with mathematical models and taking into consideration, appropriately, both the elements of the extra-mathematical domain and the progress of the phases of the modelling cycle (NISS; HØJGAARD, 2019).

SELF-REGULATION

Self-regulation is a transversal competency, fundamental for the learning of mathematics, which provides students with a series of tools to conduct a self-regulated learning.

The latter is defined as that active process through which students set learning objectives, and then monitor, regulate, and control their cognition, motivation, and behaviour, guided by those goals and contextual aspects (PINTRICH, 2004). This learning can be promoted in the teaching of mathematics through various teaching practices, which are related to the aspects to consider in the didactic design made by a teacher. These practices allow both the teacher and the student to develop themselves autonomously; be able to plan the time and means available to teach or learn; improve or keep motivation; and overcome difficulties, among others.

In general terms, a practice is understood as those actions that teachers can perform in a teaching and learning process. More specifically, in this study, we adopt the position of Hidalgo-Moncada et al. (2023), who define a self-regulation practice as “any action taken by the teacher to guide students towards self-regulated learning in mathematics” (p. 121, authors’ translation). Since these practices are part of the mathematics teachers professional work, it is important that they consider explicitly promoting self-regulated learning when teaching their discipline (ROSÁRIO et al., 2007).

METHODOLOGY

This study is of a reflective-on-theory nature, where we propose an articulation between two theoretical approaches: modelling as a mathematical competency and process, and self-regulation as a transversal competency, both applied to the analysis of the mathematical activity in modelling. To this end, we followed a methodology similar to that used in the articulations of theoretical frameworks of different levels (for example, LEDEZMA; FONT; SALA, 2023; RODRÍGUEZ-NIETO et al., 2022; among others) which, in turn, is based on the general methodology of Networking of Theories proposed by Bikner-Ahsbahs and Prediger (2014), and which we describe in this section.

In a *first step*, we developed a mutual explanation between the authors about the two theoretical frameworks, with the purpose of having a clear understanding of both, due to the specialities of each author. More specifically, the first author is a specialist in modelling, the second author in self-regulation, and the third author in Networking of Theories. This mutual explanation implied a philosophical discussion on the constitutive elements of a theory (in terms of RADFORD, 2008), namely, principles, methods, and paradigmatic research questions. Furthermore, we considered relevant to compare the position of each theoretical reference on modelling in mathematical teaching and learning processes. This comparison is synthesised in Box 1.

Box 1 – Comparison between the theoretical constitutive elements of mathematical modelling and self-regulation

Constitutive elements	**Mathematical modelling**	**Self-Regulation**
Principles	Realistic-empiricist position (see BORROMEO FERRI, 2018; KAISER-MESSMER, 1993).	Socio-cognitivist position (see CHAVES-BARBOZA; RODRÍGUEZ-MIRANDA, 2017; PANADERO; ALONSO-TAPIA, 2014).

Constitutive elements	Mathematical modelling	Self-Regulation
Methods	Analysis of the activity performed by an individual from the position of another observer-individual who knows the solving procedure of a certain modelling problem and the self-regulation practices involved in such solving procedure.	
Paradigmatic research questions	– Study of modelling competencies (see CEVIKBAS et al., 2022; GEIGER; FREJD, 2015; SCHUKAJLOW et al., 2023). – Development, optimisation, and assessment of mathematical modelling in initial and continuous teacher education (see BORROMEO FERRI, 2018).	– Analysis of the promotion of self-regulation in teachers (see DAURA, 2011; DELFINO et al., 2010; MERCHAN-RANGEL; HERNÁNDEZ-FLÓREZ, 2018). – Teaching of self-regulation (see BILIMÓRIA; ALMEIDA, 2008; CLEARY et al., 2017).
Modelling process	Proposal to characterise this process known as modelling cycle.	Modelling works with activities rich in mathematical processes, such as generalisation, changes of representation, and conjectures, through the connection with other disciplines and individuals' daily life.

Source: Authors' elaboration

This *first step* was essential for continuing with the following ones since, although the paradigmatic research questions are of different nature, due to the research focus of each theoretical frameworks, the concordances made evident between both approaches motivated us to continue with the following *steps*.

In a *second step*, the first author proposed a modelling problem as a context for reflection, namely, the *Lighthouse Problem* (see Figure 2). From this problem, this author developed a solving protocol, which can be considered as the expert solving procedure since it is a paradigmatic problem to explain the modelling cycle (see BLUM; BORROMEO FERRI, 2009), based on the phases and transitions of the MMCCP (see Figure 1), which is described below.

Figure 2 – The Lighthouse Problem

Lighthouse

In the bay of the city of Bremen, a lighthouse measuring 30,7 m and called “Red Sand” was built directly on the coast in 1884. With its beacon, it was meant to warn ships that they were approaching the coast.

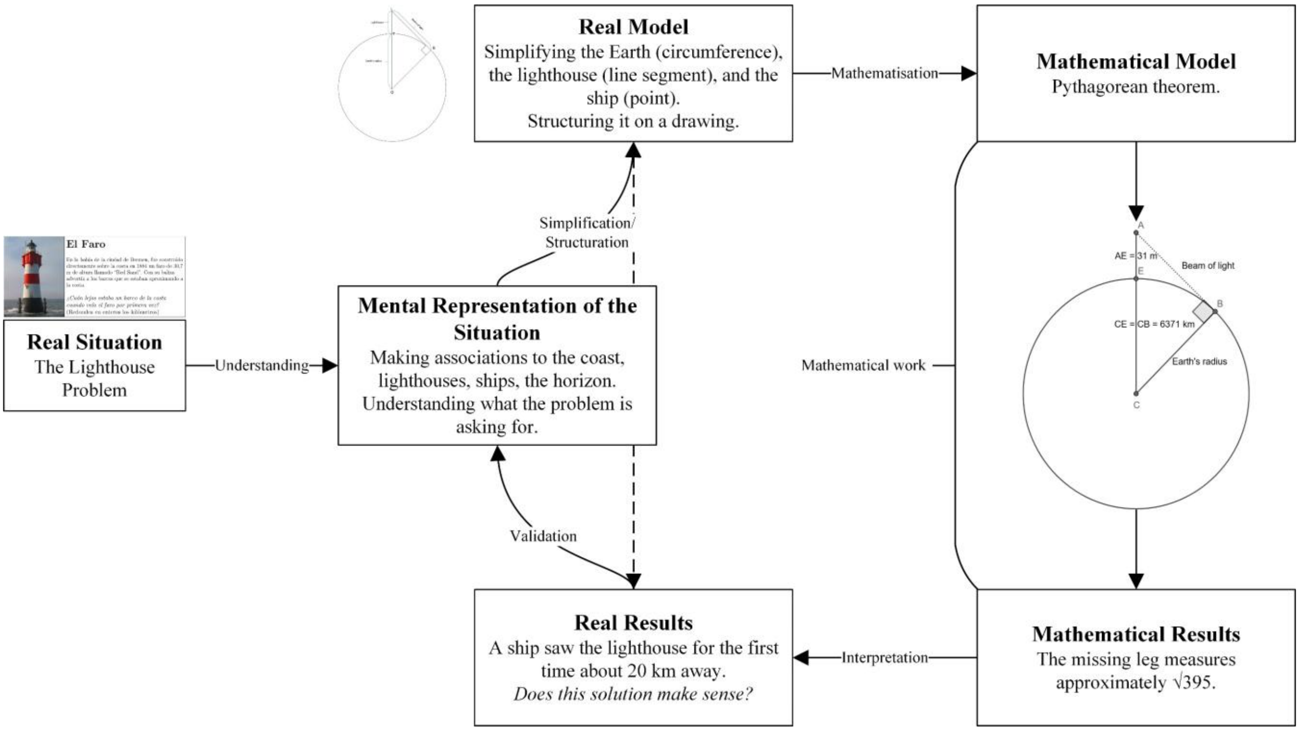
How far was a ship still away from the coast when the lighthouse could be seen for the first time? (Round up to full kilometres)

Source: Adapted from Blum and Borromeo Ferri (2009, p. 48)

The *real situation* is understood as a problem-situation taken from reality, that is, the *Lighthouse Problem* through a picture. From this, the individual forms a *mental representation of the situation* in which he understands the task and mentally reconstructs the situation, therefore, he relates the wording of the task to the coast and his own experiences with lighthouses (*extra-mathematical knowledge*) and understands that he must determine the distance a ship was when it first saw the lighthouse. To construct a *real model*, the individual must, on one side, simplify the mental image that has been formed, so the Earth can be simplified as a circumference, the lighthouse as a line segment, and the ship as a point on the circumference; and, on the other side, structure it through a representation. The *mathematical model* takes into consideration the mathematical objects that allow the *real situation* to be explained (ABASSIAN et al., 2020), and will be the product of the mathematisation (translation into mathematical language) of the *real model* and the contributions of the individual’s *extra-mathematical knowledge*. In the case of the *Lighthouse Problem*, the Pythagorean theorem can be used as a *mathematical model*. From working with the *mathematical model*, *mathematical results* are obtained that, in this problem, would be approximately $\sqrt{395}$. When these *mathematical results* are interpreted in the context of the *real situation*, they will lead to obtaining *real results* (validated by comparing the *real results* ↔ *mental representation of the situation* ↔ *real model* triad), which would lead to a plausible answer for the proposed problem, in this case, an approximate distance of 20 km.

The expert solving described above can be represented, in terms of the MMCCP, using the scheme in Figure 3.

Figure 3 – Solving scheme for the *Lighthouse Problem* using the MMCCP.



Source: Authors' elaboration.

In a *third step*, the second author took the expert solving of the problem in Figure 2 and identified the practices that promote self-regulated learning that could be present in solving this problem. To this end, she related one (or more) of these practices to some transitions of the MMCCP, based on the proposal by Hidalgo-Moncada et al. (2023). The fact of focusing on the transitions and not on the phases of the MMCCP is justified because it is in these transitions where the mathematical activity of the modelling process occurs, meanwhile the phases can be considered as inputs/outputs of a portion of mathematical activity in this process (LEDEZMA; FONT; SALA, 2023).

This relationship between the transitions of the MMCCP and the practices that promote self-regulated learning is presented in Box 2.

Box 2 – Practices that promote self-regulated learning in the MMCCP.

Transitions of the MMCCP	**Practices that promote self-regulated learning***
Simplification/Structuration	– Link the study of mathematical contents to the environment and daily life. Show interdisciplinary connections.
Mathematisation	– Teach students to check their understanding of mathematical contents. – Propose activities in which students must generalise a formula, make intra-mathematical connections, changes of representation, conjectures, etc.

Transitions of the MMCCP	Practices that promote self-regulated learning*
Mathematical work	– Propose the search for and comparison of different solutions for the same problem.
Interpretation of results	– Promote students' identification of the errors that they made, their causes, and how to avoid them.
Validation of results	– Promote the argumentation and explanation of the procedures used.

Note (*): Practices extracted from Hidalgo-Moncada *et al.* (2023). Source: Authors' elaboration

Up to this point of the study, we the authors had only reflected on the modelling problem from a theoretical point of view, starting by a philosophical discussion on the nature of both frameworks, continuing with the expert solving procedure of a paradigmatic modelling problem, and ending by the identification of the practices that promote self-regulated learning in the different transitions of the MMCCP. However, we considered it pertinent to test these reflections in an implementation context with mathematics teachers.

In a *fourth step*, we conducted the implementation phase of our study. To this end, we designed a workshop aimed at practising secondary education mathematics teachers, whose objective was to introduce participants to the importance of self-regulated learning in solving modelling problems, and where we posed the *Lighthouse Problem* (Figure 2). This workshop was implemented in the context of an international academic event in which we the authors participated. This workshop was structured in two sessions, developed in virtual mode (by provisions of the event's organising committee), as follows: in the first session, we introduced the participants to general aspects of modelling, we posed the problem in question to them for its solving, and we asked some reflection questions about it from the perspective of self-regulated learning; in the second session, we continued with a similar work dynamic, although with another modelling problem, which we do not analyse in this article since it exceeds our interests. Therefore, the participation of the additional study subjects was voluntary, based on their interest in the workshop topic and their informed consent as part of this study.

Some of the questions were directed at the participants as solvers of the modelling problem (questions 1–4) and others were directed at them in their role as teachers (questions 5–7). Furthermore, after the reflection made by the authors during the *third step*, we decided to incorporate two new questions related to certain practices that promote self-regulated learning that, although they do not correspond to any specific phase or transition of the MMCCP, they are implicitly present in the solving of a modelling problem. The first of these questions refers to considering the interests of the solvers to increase their motivation (question 3), and the second question is related to the incorporation of ICT elements (questions 7). The questions posed during the workshop are presented in Box 3, along with the corresponding practice related to each question.

Box 3 – Reflection questions posed during the workshop

Questions	**Practices***
(1) What mathematical concepts are involved in solving this problem? What intra- and extra-mathematical considerations should be taken into account?	– Teach students to check their understanding of mathematical contents. – Propose activities in which students must generalise a formula, make intra-mathematical connections, changes of representation, conjectures, etc. – Link the study of mathematical contents to the environment and daily life. Show interdisciplinary connections.
(2) Are there different solutions to this problem? If yes, what other way(s) to solve the problem is(are) there?	– Propose the search for and comparison of different solutions for the same problem.
(3) Do you think this problem met your interests? What changes could be applied to the problem to make it more attractive and motivating?	– Consider the interests of the students, their family and social context to generate activities related to their interests, allowing a better emotional, motivational, and attitudinal behaviour.
(4) What links can be observed between mathematics and your own environment in this problem?	– Link the study of mathematical contents to the environment and daily life. Show interdisciplinary connections.
(5) What errors do you think your students would make when solving this problem? Do you think it is important to discuss the errors with your students? Why?	– Promote students' identification of errors made, their causes, and how to avoid them.
(6) Is it possible to encourage argumentation in your students with this type of problems? Why?	– Promote the argumentation and explanation of the procedures used.
(7) Could ICT or other types of materials be used to present and solve the problem? If yes, could you mention some of them? What would be their benefits?	– Implement different teaching means that enhance the search for, processing, and obtaining information that students must assimilate, which will help the understanding of mathematical concepts, tasks, or activities.

Note (*): Practices extracted from Hidalgo-Moncada et al. (2023). Source: Authors' elaboration

In a *fifth step*, developed jointly by the three authors, we analysed the participants' solving procedures and their responses to the reflection questions about the *Lighthouse Problem*. To this end, we reviewed the video recording of the first session of the workshop described in the *fourth step*; then, we transcribed the respective dialogues; finally, we conducted the analysis based on the proposed solving scheme (see Figure 3) and the practices that promote self-regulated learning in the MMCCP (see Box 2).

Finally, in a *sixth step*, we corroborated the proposal made in the *third step* with the results of the *fifth step* to refine the articulation between both theoretical approaches.

PRESENTATION AND ANALYSIS OF RESULTS

In this section, we jointly present and analyse the solving procedures of the participating teachers to the *Lighthouse Problem* and their responses to the reflection questions during the first session of the workshop, based on the transcriptions of their interventions. To identify the participants, we labelled them T*x*, where *x* is a distinctive number for each. Although 10 teachers participated in our workshop, we highlight the intervention of 5 of them, as they were those who interacted the most during the session.

In accordance with the dynamics established for the workshop, we decided to give the participants 20 minutes to solve the problem in Figure 2 and record their answers through a link to the Mentimeter platform. After this time, we posed two sets of questions to the participants from the perspective of self-regulated learning, as detailed in Box 3. Although the participants did not record their answers on the Mentimeter platform, they still explained part of their solving procedures and answers to the problem during the reflection questions.

The first four questions were aimed at making the participants assume the role of student problem-solvers and reflecting on it. In the following paragraphs, we present and analyse some of the responses given to questions 1–4.

(1) What mathematical concepts are involved in solving this problem? What intra- and extra-mathematical considerations should be taken into account?

To respond, the participants had another link to the Mentimeter platform intended for question (1), recording concepts such as: geometry, trigonometric functions, length, arithmetic, triangles, distance, cartesian plane, among others. After this, the participants gave the following opinions:

T1) *You should take into consideration the shape of the Earth, know how a lighthouse works*.

T2) *The angle of observation of a lighthouse or the radius of the Earth, among others, as possible extra-mathematical connections*.

Discussing the concepts involved in solving a modelling problem allows to autonomously check the understanding of the mathematical contents involved in the problem. For its part, observing intra-mathematical connections will lead students to acquire proposed mathematical contents in a deeper way. Likewise, discussing extra-mathematical connections will allow students to feel more connected and closer to what is intended to be taught, which will lead to an increase in motivation when learning mathematics.

In terms of the MMCCP, the responses of the participants to question (1) are related to the *mental representation of the situation* phase, by establishing associations between their experiences and the context of the problem; the *real model* phase, when developing the first representation with the information provided by the wording of the problem and its *extra-mathematical knowledge*; and the *mathematical model* phase, since mathematisation requires the establishment of connections between different intra-mathematical contents.

(2) Are there different solutions to this problem? If yes, what other way(s) to solve the problem is(are) there?

The responses of the participants to question (2) were as follows:

T2) *One way to solve would be using trigonometry, calculating the angle of depression of the lighthouse*.

T3) *Another way could be with spherical geometry, delving into curved shapes*.

Discussing the different methods of solving a modelling problem allows students to understand it in greater detail. In addition, it is an opportunity to carry out peer learning and develop argumentative skills through debate.

In terms of the MMCCP, the responses of the participants to question (2) are related to the *mathematical model* phase and its subsequent mathematical work, since they describe their solving procedures to obtain *mathematical results*. Although the participants did not explicitly detail how they solved it, the description of the *real model* was somewhat related to our proposal in Figure 3.

(3) Do you think this problem met your interests? What changes could be applied to the problem to make it more attractive and motivating?

The responses of the participants to question (3) were as follows:

T1) *I would change the lighthouse for a torch with an object or shadow.*

T2) *Adapt it to a scenario accessible to students.*

To answer this question, the participants did not refer to their interests per se, but rather thought about adapting it to a context close to their students. However, what this question sought was to reflect on the importance of considering the interests of those who solve the problem, so that it is motivating for them, and they face it with a positive attitude. In other words, the participants could not detach themselves from their role as teachers and place themselves in the role of the student problem-solver.

(4) What links can be observed between mathematics and your own environment in this problem?

The responses of the participants to question (4) were as follows:

T4) *Although it is not from my environment, I can associate it with what I work, with terrain measurements, geographic information systems in which they already include the curvature of the Earth.*

T5) *It has no link with my environment.*

T3) *Yes, it is related to my environment, since I live in Lima, which is a coastal city, and there is a district that has a lighthouse that is on a hill, so I tried to couple this knowledge with the wording of the problem to be able to solve it.*

T1) *I can relate it to the lightning of a lamp or a torch.*

This question sought to make the participants reflect on the importance of considering contexts close to those who must solve the modelling problem in question, given that, in this way, they will be able to associate it with previous mathematical knowledge, thus facilitating the understanding of the problem.

In terms of the MMCCP, the responses of the participants to question (4) are explicitly related to the extra-mathematical considerations that must be made to begin solving a modelling problem.

The last three questions were aimed at making the participants assume the role of teachers who pose the problem to their students. In the following paragraphs, we present and analyse some of the responses given to questions 5–7.

(5) What errors do you think your students would make when solving this problem? Do you think it is important to discuss the errors with your students? Why?

The responses of the participants to question (5) were as follows:

T1) *Students might think that curvature is negligible. It is important to work with errors, since we learn from them, even as teachers.*

T2) *Students often mix up units of length, such as metres and kilometres.*

T3) *They may say that there is no solution.*

T4) *They could say that the problem lacks information*.

This question allowed the participants not to forget that errors are a good method or instance of learning, in addition to showing them that this allows students to develop other skills, such as argumentation and debate. An aspect to highlight from T4's response is the relationship that can be established between the 'understanding of the task' transition of the MMCCP (see Figure 1, no. 1), and the characteristics of a problem of this type, which – among others – must be open and complex. In this sense, the wording of the problem must be challenging for the solver, which does not mean that it is impossible to solve.

In terms of the MMCCP, the responses of the participants to question (5) are related to the interpretation of *mathematical results* into *real results*, since it is the transition in which the problem is once again placed in the real world. Furthermore, verification of procedural errors during mathematical work can shed light on whether it is necessary to revise other phases of the MMCCP or repeat the entire cycle.

(6) Is it possible to encourage argumentation in your students with this type of problems? Why?

The responses of the participants to question (6) were as follows:

T5) *Argumentation will depend on how the teacher manages the environment, compared to the students' responses*.

T2) *When explaining why it has no solution, students would use mathematical concepts and their relationship*.

T3) *In a scenario that allows students to investigate about lighthouses and there build the argument and solution to the problem. In this way, the strategies of others could be analysed and evaluated*.

This question sought to enable the participants to imagine the various ways in which argumentation can be encouraged with modelling problems and other skills that can be developed from argumentation, such as debate.

In terms of the MMCCP, the responses of the participants to question (6) are related to the validation of *real results* in the initial context of the problem, and where the arguments about the plausibility (or not) of these results becomes highly relevant.

(7) Could ICT or other types of materials be used to present and solve the problem? If yes, could you mention some of them? What would be their benefits?

The responses of the participants to question (7) were as follows:

T1) *You can use some simulation in GeoGebra or other dynamic-geometry software to observe the curvature and begin to solve the problem*.

This question allowed the participants to observe that the use of ICT can facilitate both the explanation of this modelling problem and its solving. In addition, it allows students to work autonomously, as well as increase their motivation for mathematics.

DISCUSSION AND CONCLUSIONS

In this article, we proposed an articulation between two theoretical approaches: on one hand, modelling as a mathematical competency and process; on the other hand, self-regulation as a transversal competency, specifically, from the development of self-regulated learning; both applied to the analysis of the mathematical activity in modelling.

On a theoretical level, we the authors started by a philosophical discussion that allowed us to compare the constitutive elements of each theoretical framework (see Box 1).

In this comparison, we could make evident a coincidence regarding the method to analyse the mathematical activity performed by an individual when solving a problem (in this case, a modelling problem), which consists of the observation, on the part of a researcher, who already knows both the procedure to solve the problem and the self-regulation practices involved in such solving procedure. This coincidence was a key to propose a first articulation between both approaches based on the expert solving of a modelling problem, by identifying practices that promote self-regulated learning in some of the transitions of the MMCCP (see Box 2). Then, this first proposal was complemented with the design of a practical-reflective workshop for teachers, where the *Lighthouse Problem* was posed with reflection questions about its solving (see Box 3). The dynamics of this workshop made it possible for participants to solve a modelling problem, learn about a tool to analyse this process (the MMCCP in Figure 1), and reflect on the incorporation and implementation of various practices that allow promoting self-regulated learning when working with modelling problems.

The practices that promote self-regulated learning are not explicitly included in the MMCCP, however, we can conclude that there are some of these that are present throughout the modelling process and that should be considered, not only during the solving of this type of problems, but also when designing mathematical teaching and learning processes that aim to work on modelling in the classroom. Therefore, the idea of Kistner et al. (2010) on the importance of explicitly stimulating practices that promote self-regulated learning is reinforced. More specifically, the contributions of articulating both theoretical approaches to analyse the mathematical activity in modelling can be distinguished from the teachers' and students' perspectives.

From the teachers' perspective, incorporating these practices when planning a mathematical teaching and learning process that includes modelling would allow them, firstly, to choose (or design) a modelling problem that considers, for example, the interests (supported by MARTÍNEZ; VALIENTE, 2019), environment, and daily life of their students. Secondly, that the problem in question allows the establishment of interdisciplinary connections to enrich the understanding of the *mathematical model* of the problem, in order to extrapolate it to extra-mathematical contexts. And thirdly, that this planning considers the available time and means (in terms of HIDALGO-MONCADA et al., 2020) to develop modelling in the classroom.

From this same perspective, the incorporation of these practices when implementing this mathematical teaching and learning process would allow teachers to anticipate the difficulties that may emerge in their students during the modelling process, by considering, for example, the errors or certain blockages (in terms of GALBRAITH; STILLMAN, 2006) during the different phases and transitions of the modelling cycle, the understanding of the results given different solving paths for the problem, among others. These considerations are related to the cyclical nature of the modelling process, that is, to the need to resume the work performed in previous phases, or the reiteration of the entire cycle, when errors occur, following different modelling routes (in terms of BORROMEO FERRI, 2010), using complementary means (graphical software, information from other sources), or other strategies (restructuring the *real model* of the *mathematical model*). Particularly, the use of ICT is not explicitly considered by the MMCCP, so it is an important aspect to take into account when designing and implementing modelling in the classroom.

Finally, the incorporation of these practices would encourage the development of complementary processes to modelling, such as argumentation (see TEKIN, 2019), the establishment of intra- and extra-mathematical connections, changes in representation, formulation of conjectures, etc., which are not only required during the work with the *mathematical model*, but also in the different phases and transitions of the MMCCP.

From the students' perspective, incorporating these practices when facing a modelling problem would allow them to have greater tools that transcend mathematics, and that would allow them to acquire self-regulation competency, specifically, in mathematical modelling. More specifically, students who manage to acquire self-regulation competency can improve their learning and academic performance (SANMARTÍ, 2010), in the same way that it leads them to be autonomous individuals, not only in the mathematics classroom, but also throughout their lives.

For example, literature recommends that work with modelling in the classroom be developed in groups of students (see ENGLISH, 2003; LESH; DOERR, 2003), and one of the practices that promote self-regulated learning points in this direction when developing cooperative work. This group work could also be a driver of discussion among peers and, therefore, of argumentation. On the other hand, since modelling problems are characterised by being realistic and authentic situations (in terms of PALM, 2007), if teachers adapt these two attributes to the environment or interests of their students, they could feel more involved with the problem and, in addition, their motivation in learning mathematics would improve.

Resuming the research question of our study on what is the role played by self-regulation practices in the mathematical modelling process, we can affirm that, although various authors have individually addressed the different aspect mentioned above in studies about modelling (errors, mathematical processes, work dynamics, etc.), it is interesting to have a unified proposal that integrates these elements with the MMCCP, such as the proposal that we raised in this article. More specifically, in Box 2 we established incipient relationships between the MMCCP and practices that promote self-regulated learning, based on a self-regulation instrument (developed by one of the authors in previous studies) applied to this modelling cycle; in Box 3 we added questions that emerged from other practices not explicitly considered in the MMCCP; and the results made it possible to evidence the presence of other practices not considered by the authors. For example, the practices 'Propose activities where discussion among peers is encouraged'; 'Develop cooperative work'; 'Describe the way of reasoning when solving a problem, supporting students in their attempt to understand the problems that they develop individually' emerged from the results reported in this study. In other words, incorporating practices that promote self-regulated learning allows teachers to be promoters of autonomy, critical thinking, and a reflective attitude in their students.

However, one of the limitations of our study was that, due to the virtual modality of the workshop, we did not have a large number of participants who interacted with each other and enabled the emergence of possible new practices that promote self-regulated learning, in addition to those considered for the design of the workshop (see Box 3) during the solving of a modelling problem. Therefore, we hope to implement this experience again in a more favourable context to the interest of our study and taking into consideration the importance of modelling in the education of prospective mathematics teachers (see a broader discussion in LEDEZMA; BREDA; FONT, 2023).

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REFERENCES

ABASSIAN, A.; SAFI, F.; BUSH, S.; BOSTIC, J. Five different perspectives on mathematical modeling in mathematics education. **Investigations in Mathematics Learning**, Greensboro, v. 12, n. 1, p. 53–65. 2020. DOI: 10.1080/19477503.2019.1595360.

ALTUN, S.; ERDEN, M. Self-regulation-based learning strategies and self-efficacy perceptions as predictors of male and female students' mathematics achievement. **Procedia – Social and Behavorial Sciences**, Netherlands, v. 106, p. 2354–2364, Dec. 2013. DOI: 10.1016/j.sbspro.2013.12.270.

BIKNER-AHSBAHS, A.; PREDIGER, S. (ed.). **Networking of Theories as a Research Practice in Mathematics Education**. Cham: Springer, 2014. 329 p. DOI: 10.1007/978-3-319-05389-9.

BILIMÓRIA, H.; ALMEIDA, L. S. Aprendizagem auto-regulada: Fundamentos e organização do Programa SABER. **Psicologia Escolar e Educacional**, São Paulo, v. 12, n. 1, p. 13–22, Jun. 2008. DOI: 10.1590/S1413-85572008000100002.

BLUM, W. Can modelling be taught and learnt? Some answers from empirical research. In: KAISER, G.; BLUM, W.; BORROMEO FERRI, R.; STILLMAN, G. (ed.). **Trends in Teaching and Learning of Mathematical Modelling:** ICTMA 14. Dordrecht: Springer, 2011. p. 15–30. DOI: 10.1007/978-94-007-0910-2_3.

BLUM, W.; BORROMEO FERRI, R. Mathematical modelling: Can it be taught and learnt? **Journal of Mathematical Modelling and Application**, Blumenau, v. 1, n. 1, p. 45–58, Dec. 2009.

BORJI, V.; FONT, V.; ALAMOLHODAEI, H.; SÁNCHEZ, A. Application of the complementarities of two theories, APOS and OSA, for the analysis of the university students' understanding on the graph of the function and its derivative. **Eurasia Journal of Mathematics, Science and Technology Education**, Eastbourne, v. 14, n. 6, p. 2301–2315, Jun. 2018. DOI: 10.29333/ejmste/89514.

BORROMEO FERRI, R. Theoretical and empirical differentiations of phases in the modelling process. **Zentralblatt für Didaktik der Mathematik**, Berlin, v. 38, n. 2, p. 86–95, Apr. 2006. DOI: 10.1007/bf02655883.

BORROMEO FERRI, R. On the influence of mathematical thinking styles on learners' modeling behavior. **Journal für Mathematik-Didaktik**, v. 31, n. 1, p. 99–118, Mar. 2010. DOI: 10.1007/s13138-010-0009-8.

BORROMEO FERRI, R. **Learning How to Teach Mathematical Modeling in School and Teacher Education**. Cham: Springer, 2018. 163 p. DOI: 10.1007/978-3-319-68072-9.

CEVIKBAS, M.; KAISER, G.; SCHUKAJLOW, S. A systematic literature review of the current discussion on mathematical modelling competencies: State-of-the-art developments in conceptualizing, measuring, and fostering.

Educational Studies in Mathematics, Dordrecht, v. 109, n. 2, p. 205–236, Feb. 2022. DOI: 10.1007/s10649-021-10104-6.

CHAVES-BARBOZA, E.; RODRÍGUEZ-MIRANDA, L. Aprendizaje autorregulado en la teoría sociocognitiva: Marco conceptual y posibles líneas de investigación. **Revista Ensayos Pedagógicos**, Heredia, v. 12, n. 2, p. 47–71, Jul.–Dec. 2017. DOI: 10.15359/rep.12-2.3.

CLEARY, T. J.; VELARDI, B.; SCHNAIDMAN, B. Effects of the Self-Regulation Empowerment Program (SREP) on middle school students' strategic skills, self-efficacy, and mathematics achievement. **Journal of School Psychology**, Madison, v. 64, p. 28–42, Oct. 2017. DOI: 10.1016/j.jsp.2017.04.004.

CUELI, M.; GARCÍA, T.; GONZÁLEZ-CASTRO, P. Autorregulación y rendimiento académico en matemáticas. **Aula Abierta**, Oviedo, v. 41, n. 1, p. 39–48, Jan. 2013.

DAURA, F. T. La asesoría académica universitaria: Un espacio propicio para la promoción del aprendizaje autorregulado. **Revista de Orientación Educacional**, Valparaíso, v. 25, n. 47, p. 49–63. 2011.

DELFINO, M.; DETTORI, G.; PERSICO, D. An online course fostering self-regulation of trainee teachers. **Psicothema**, Oviedo, v. 22, n. 2, p. 299–305. 2010.

ENGLISH, L. Mathematical modelling with young learners. In: LAMON, S. J.; PARKER, W. A.; HOUSTON, K. (ed.). **Mathematical Modelling:** A Way of Life – ICTMA 11. Chichester: Horwood, 2003. p. 3–17.

FREJD, P.; VOS, P. The spirit of mathematical modeling – A philosophical study on the occasion of 50 years of mathematical modeling education. **The Mathematics Enthusiast**, Missoula, v. 21, n. 1–2, p. 269–300, Feb. 2024. DOI: 10.54870/1551-3440.1626.

GEIGER, V.; FREJD, P. A reflection on mathematical modelling and applications as a field of research: Theoretical orientation and diversity. In: STILLMAN, G.; BLUM, W.; BIEMBENGUT, M. S. (ed.). **Mathematical Modelling in Education Research and Practice:** Cultural, Social and Cognitive Influences. Cham: Springer, 2015. p. 161–171. DOI: 10.1007/978-3-319-18272-8_12.

GALBRAITH, P.; STILLMAN, G. A framework for identifying student blockages during transitions in the modelling process. **Zentralblatt für Didaktik der Mathematik**, Berlin, v. 38, n. 2, p. 143–162, Apr. 2006. DOI: 10.1007/bf02655886.

HIDALGO-MONCADA, D.; DÍEZ-PALOMAR, J.; VANEGAS, Y. Formación de maestros de educación primaria en el contexto de confinamiento. La importancia del aprendizaje autorregulado en las matemáticas. **Magister: Revista de Formación del Profesorado e Innovación Educativa**, Oviedo, v. 32, n. 1, p. 40–48, Sept. 2020. DOI: 10.17811/msg.32.1.2020.40-48.

HIDALGO-MONCADA, D.; DÍEZ-PALOMAR, J.; VANEGAS, Y. Prácticas de autorregulación en la propuesta didáctica de un futuro profesor de matemáticas: Un instrumento para la reflexión. **PARADIGMA**, Maracay, v. XLIV, n. 2, p. 112–146, Jun. 2023. DOI: 10.37618/PARADIGMA.1011-2251.2023.p112-146.id1384.

KAISER, G. Mathematical modelling and applications in education. In: LERMAN, S. (ed.). **Encyclopedia of Mathematics Education**. 2. ed. Cham: Springer, 2020. p. 553–561. DOI: 10.1007/978-3-030-15789-0_101.

KAISER-MESSMER, G. Reflection on future developments in the light of empirical research. In: BREITEIG, T.; HUNTLEY, I.; KAISER-MESSMER, G. (ed.). **Teaching and Learning Mathematics in Context**. Chichester: Ellis Horwood, 1993. p. 213–218.

KISTNER, S. et al. Promotion of self-regulated learning in classrooms: Investigating frequency, quality, and consequences for student performance. **Metacognition and Learning**, United States, v. 5, n. 2, p. 157–171, Aug. 2010. DOI: 10.1007/s11409-010-9055-3.

LAVASANI, M. G.; MIRHOSSEINI, F. S.; HEJAZI, E.; DAVOODI, M. The effect of self-regulation learning strategies training on the academic motivation and self-efficacy. **Procedia – Social and Behavioral Sciences**, Netherlands, v. 29, p. 627–632. 2011. DOI: 10.1016/j.sbspro.2011.11.285.

LEDEZMA, C.; BREDA, A.; FONT, V. Prospective teachers' reflections on the inclusion of mathematical modelling during the transition period between the face-to-face and virtual teaching contexts. **International Journal of Science and Mathematics Education**, Taiwan, advance online publication, Aug. 2023. DOI: 10.1007/s10763-023-10412-8.

LEDEZMA, C.; FONT, V.; SALA, G. Analysing the mathematical activity in a modelling process from the cognitive and onto-semiotic perspectives. **Mathematics Education Research Journal**, Sydney, v. 35, n. 4, p. 715–741, Dec. 2023. DOI: 10.1007/s13394-022-00411-3.

LESH, R.; DOERR, H. M. Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In: LESH, R.; DOERR, H. M. (ed.). **Beyond Constructivism:** Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching. Mahwah: Lawrence Erlbaum, 2003. p. 3–33.

MAASS, K. et al. Mathematical modelling – A key to citizenship education. In: BUCHHOLTZ, N.; SCHWARZ, B.; VORHÖLTER, K. (ed.). **Initiationen mathematikdidaktischer Forschung:** Festschrift zum 70. Geburtstag von Gabriele Kaiser. Wiesbaden: Springer Spektrum, 2022. p. 31–50. DOI: 10.1007/978-3-658-36766-4_2.

MARTÍNEZ, M.; VALIENTE, C. Autorregulación afectivo-motivacional, resolución de problemas y rendimiento matemático en educación primaria. **Educatio Siglo XXI**, Murcia, v. 37, n. 3, p. 33–54, Nov.–Feb. 2019. DOI: 10.6018/educatio.399151.

MERCHAN-RANGEL, N.; HERNÁNDEZ-FLÓREZ, N. E. Rol profesoral y estrategias promotoras de autorregulación del aprendizaje en educación superior. **Revista Espacios**, Caracas, v. 39, n. 52, p. 18. 2018.

NISS, M.; HØJGAARD, T. Mathematical competencies revisited. **Educational Studies in Mathematics**, Dordrecht, v. 102, n. 1, p. 9–28, Sept. 2019. DOI: 10.1007/s10649-019-09903-9.

PALM, T. Features and impact of the authenticity of applied mathematical school tasks. In: BLUM, W.; GALBRAITH, P. L.; HENN, H.-W.; NISS, M. (ed.). **Modelling and Applications in Mathematics Education:** The 14th ICMI Study. Boston: Springer, 2007. p. 201–208. DOI: 10.1007/978-0-387-29822-1_20.

PANADERO, E.; ALONSO-TAPIA, J. Teorías de autorregulación educativa: Una comparación y reflexión teórica. **Psicología Educativa**, Madrid, v. 20, n. 1, p. 11–22. 2014. DOI: 10.1016/j.pse.2014.05.002.

PINTRICH, P. R. A conceptual framework for assessing motivation and self-regulated learning in college students. **Educational Psychology Review**, Dordrecht, v. 16, n. 4, p. 385–407, Dec. 2004. DOI: 10.1007/s10648-004-0006-x.

PRECIADO, A.; PEÑA, F.; ORTIZ, Y.; SOLARES, A. Diversity of perspectives on mathematical modelling: A review of the international landscape. In: GREEFRATH, G.; CARREIRA, S.; STILLMAN, G. A. (ed.). **Advancing and Consolidating Mathematical Modelling:** Research from ICME-14. Cham: Springer, 2023. p. 43–57. DOI: 10.1007/978-3-031-27115-1_3.

RADFORD, L. Connecting theories in mathematics education: Challenges and possibilities. **ZDM – Mathematics Education**, Berlin, v. 40, n. 2, p. 317–327, May. 2008. DOI: 10.1007/s11858-008-0090-3.

RODRÍGUEZ-NIETO, C. A.; FONT, V.; BORJI, V.; RODRÍGUEZ-VÁSQUEZ, F. M. Mathematical connections from a networking theory between Extended Theory of Mathematical Connections and Onto-Semiotic Approach. **International Journal of Mathematical Education in Science and Technology**, London, v. 53, n. 9, p. 2364–2390. 2022. DOI: 10.1080/0020739X.2021.1875071.

ROSÁRIO, P. et al. Eficacia de un programa instruccional para la mejora de procesos y estrategias de aprendizaje en la enseñanza superior. **Psicothema**, Oviedo, v. 19, n. 3, p. 353–358. 2007.

SANMARTÍ, N. Aprender a evaluarse: Motor de todo aprendizaje. **Aula de Innovación Educativa**, Barcelona, v. 192, p. 26–29, Jun. 2010.

SCHUKAJLOW, S.; KAISER, G.; STILLMAN, G. Modeling from a cognitive perspective: Theoretical considerations and empirical contributions. **Mathematical Thinking and Learning**, Filadelfia, v. 25, n. 3, p. 259–269. 2023. DOI: 10.1080/10986065.2021.2012631.

TEKIN, A. Arguments constructed within the mathematical modelling cycle. **International Journal of Mathematical Education in Science and Technology**, London, v. 50, n. 2, p. 292–314. 2019. DOI: 10.1080/0020739X.2018.1501825.