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ARTIGO ORIGINAL/ ORIGINAL ARTICLE

Reflexiones de Futuros Profesores sobre la Inclusión de la Modelización Matemática en Tres Contextos de Enseñanza

Prospective Teachers' Reflection on the Inclusion of Mathematical Modelling in Three Teaching Contexts

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RESUMEN

En este artículo, se sintetizan y discuten los principales resultados de una investigación, a partir de dos estudios previamente desarrollados por los autores y su continuación metodológica, que abarcó tres años académicos de un programa de máster para profesores de matemática de educación secundaria y bachillerato, cuyo objetivo fue analizar los aspectos del proceso de enseñanza y aprendizaje matemático que los futuros profesores priorizaron en sus reflexiones sobre la inclusión de la modelización durante sus prácticas educativas, las cuales se desarrollaron en tres contextos de enseñanza diferentes debido a la pandemia por COVID-19 y sus repercusiones. Estas reflexiones se analizaron con el constructo Criterios de Idoneidad Didáctica, propuesto por el Enfoque Onto-Semiótico, que fue la misma herramienta utilizada por los futuros profesores para pautar la reflexión sobre su propia práctica. Se siguió una metodología de investigación cualitativa desde un paradigma interpretativo y con un enfoque naturalista, pues no se intervino en el Máster de Formación de Profesores de Matemática estudiado. Se realizó un análisis de contenido sobre 337 Trabajos Finales de Máster elaborados durante los cursos 2019–2020, 2020–2021, y 2021–2022, con prácticas educativas desarrolladas en los contextos de enseñanza virtual, semipresencial, y presencial, respectivamente. Se destacan los siguientes resultados: (a) alrededor de un tercio de los futuros profesores afirmaron que implementaron la modelización en sus unidades didácticas y reflexionaron sobre su inclusión; (b) en sus reflexiones, los futuros profesores valoraron positivamente la inclusión de la modelización basados principalmente en los criterios de idoneidad epistémico y ecológico. Se concluye con algunas reflexiones sobre el impacto de los tres contextos de enseñanza en la implementación de la modelización.

Palabras clave: Análisis de contenido, Criterios de idoneidad didáctica, Trabajo final de máster.

ABSTRACT

In this article, we synthesise and discuss the main results of a research, based on two studies previously conducted by the authors and their methodological ongoing, that covered three academic years of a master's programme for secondary and baccalaureate education mathematics teachers, whose objective was to analyse the aspects of the mathematical teaching and learning process that prospective teachers prioritised in their reflections on the inclusion of modelling during their educational internship experiences, which were developed in three different teaching contexts due to the COVID-19 pandemic and its repercussions. We analysed these reflections with the Didactic Suitability Criteria construct, proposed by the Onto-Semiotic Approach, which was the same tool used by the prospective teachers to guide the reflection on their own practice. We followed qualitative research methodology from an interpretative paradigm and with a naturalistic approach, since we did not intervene in the Master's Programme in Mathematics Teacher Education studied. We performed a content analysis on 337 Master's Degree Final Projects prepared during the academic years 2019-2020, 2020-2021, and 2021-2022, with educational internship experiences developed in virtual, hybrid, and face-to-face teaching contexts respectively. We highlight the following results: (a) around one third of the prospective teachers affirmed that they implemented modelling in their didactic units and reflected on its inclusion; (b) in their reflections, the prospective teachers positively assessed the inclusion of modelling based on the epistemic and ecological suitability criteria. We conclude with some reflections on the impact of the three teaching contexts on the implementation of modelling.

Keywords: Content analysis, Didactic suitability criteria, Master's degree final project.

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INTRODUCTION

A key aspect in teacher education is reflection on own practice as a mechanism of professional improvement (Schön, 1987). Within research in Mathematics Education, the Onto-Semiotic Approach (OSA; Godino *et al*., 2007) has broadly addressed this topic, both from teacher reflection in its entirety (see Breda, 2020; Garcés, 2021; García-Marimón *et al*., 2021; among others) and from specific processes and competencies (see Hidalgo-Moncada *et al*., 2023; Sánchez *et al.*, 2022; Sol *et al*., 2023; among others). Following this research line, in this article, we report a study focused on the reflections made by prospective teachers on the inclusion of mathematical modelling during their educational internship experiences.

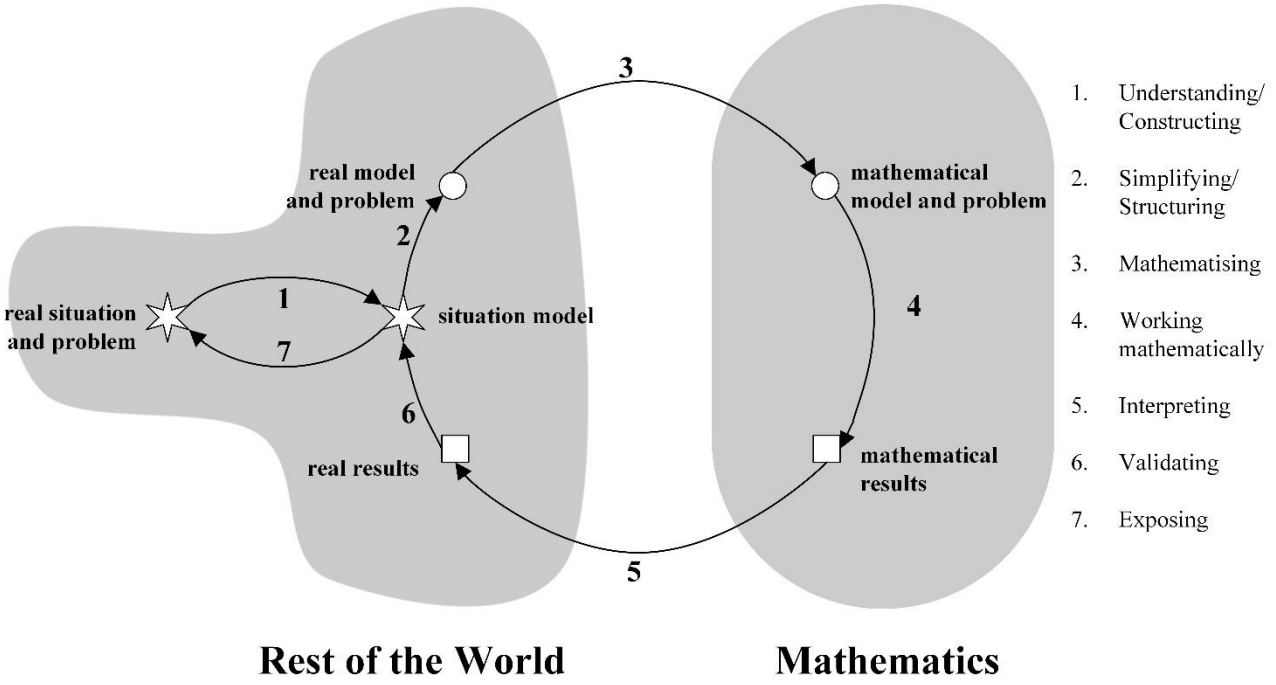
During three academic years (2019-2020, 2020-2021, and 2021-2022), we conducted research in a professionalising master's programme for secondary and baccalaureate education mathematics teachers (students aged 12-18) in the Spanish context. The main question that guided this research was: What aspects of the mathematical teaching and learning process did prospective teachers prioritise in their reflections on the inclusion of modelling during their educational internship experiences? To answer it, we analysed the Master's Degree Final Projects (MFPs) of prospective teachers, in which they mainly had to reflect on the didactic unit that they implemented during their educational internship experiences and thus propose improvements for its redesign. The prospective teachers guided their reflections using the Didactic Suitability Criteria construct (Godino, 2013), which is the tools proposed by OSA to assess the aspects (or facets) intervening in mathematical teaching and learning processes. Among these reflections, we found some that addressed the inclusion of modelling in the didactic units, and which were the focus of this research. Due to the COVID-19 pandemic and its repercussions (see a broader discussion in Engelbrecht *et al*., 2023), the prospective teachers of this master's programme had to carry out their educational internship experiences in three teaching contexts: virtual (2019-2020 course), hybrid (2020-2021 course), and face-to-face (2021-2022 course) contexts.

The relevance of this research lies in the fact that, in addition to addressing an important topic for Mathematics Education as modelling is, it does it from the perspective of teacher reflection in teacher education – using the theoretical-methodological tools provided by OSA – on a topic that this framework had not delved into before this research. Given this importance, in this article, we synthesise and discuss the main results of this research, from two studies previously conducted by the authors (see Ledezma; Breda; Font, 2024; Ledezma; Sánchez; Hidalgo-Moncada, 2024), emphasising, on one hand, the aspects of the mathematical teaching and learning process that the prospective teachers prioritised in their reflections on the inclusion of modelling during their educational internships experiences and, on the other hand, the influence of the teaching contexts on these reflections.

MATHEMATICAL MODELLING

In this research, we took as a theoretical reference the modelling cycle proposed by Blum and Leiß (2007), since it is the cycle taught to the prospective teachers of the master's programme where this research is contextualised. This cycle explains the modelling process as a transition between the «rest of the world» and «mathematics» for solving a problem-situation taken from reality, following a structure of six phases and seven transitions for which an individual would go through to solve a modelling problem (see Figure 1).

Figure 1 – Seven-step modelling cycle



Source: Adapted from Blum and Leiß (2007, p. 225)

In this research, we used the modelling cycle in Figure 1 as a theoretical basis for analysing the reflections made by the prospective teachers from two questions: What modelling tasks do the prospective teachers propose? And how did they analyse the mathematical activity underlying these tasks? These questions were broadly addressed in a previous study conducted by the authors (see Ledezma; Font; Sala-Sebastià, 2024), hence they are discussed in the Results and Analyses section.

Given that this research is framed within others based on OSA, it is worth remarking two aspects that relate this theoretical approach to modelling. Firstly, OSA considers modelling as a *hyper* or *mega process*, because it implies other more elementary mathematical *processes*, such as representation, argumentation, connections, etc. (Godino *et al.*, 2007). Secondly, in previous works, a deeper understanding of the mathematical activity underlying the modelling process has been developed from

the perspective of OSA and other relevant mathematical *processes* (see Ledezma *et al*., 2023; Ledezma; Rodríguez-Nieto; Font, 2024).

DIDACTIC SUITABILITY CRITERIA

In OSA, the Didactic Suitability Criteria construct (DSC) is proposed as a tool to assess mathematical teaching and learning processes (Godino, 2013). Their use in this research is justified by the fact that it is the same tool taught to the prospective teachers of the master's programme where this study is contextualised to guide their reflection on their own practice in the MFPs. The didactic suitability of a teaching and learning process is understood as the degree to which it (or a part of it) meets certain characteristics that allow qualifying it as suitable (optimal or adequate) for reaching the adaptation between the *personal meanings* achieved by the students (learning) and the intended or implemented *institutional meanings* (teaching), considering the circumstances and available resources (environment).

This multidimensional construct consists of six *criteria*, each focused on one specific aspect (or facet) of the mathematical teaching and learning process. In turn, each *criterion* has its own *components*, and their functionality requires defining a set of observable *indicators* that allow assessing the degree of suitability of a mathematical teaching and learning process (Breda *et al*., 2017). Box 1 presents the *components* of each *criterion* along with the codes used in this research to label them.

Box 1 – Didactic suitability criteria and their components

Criteria	Description	Components
Epistemic	For assessing whether the mathematics that is taught is a *good mathematics*.	– Errors (ES1). – Ambiguities (ES2). – Richness of processes (ES3). – Representativeness of the complexity of the mathematical object (ES4).
Cognitive	For assessing, before starting the teaching and learning process, whether what is intended to be taught is at a reasonable distance from what students know; and after, whether students learnt what was intended to be taught.	– Prior knowledge (CS1). – Curricular adaptation to individual differences (CS2). – Learning (CS3). – High cognitive demand (CS4).
Interactional	For assessing whether the interaction solves the students' doubts and difficulties.	– Teacher-student interaction (IS1). – Student interaction (IS2). – Autonomy (IS3).

Criteria	Description	Components
		– Formative assessment (IS4).
Mediational	For assessing the adequacy of material and time resources used in the teaching and learning process.	– Material resources (MS1). – Number of students, schedule, and classroom conditions (MS2). – Time (MS3).
Affective	For assessing the students' involvement (interest, motivation) in the teaching and learning process.	– Interests and needs (AS1). – Attitudes (AS2). – Emotions (AS3).
Ecological	For assessing the adequacy of the teaching and learning process to the school's educational project, the curricular guidelines, the conditions of the social and professional environment, etc.	– Adaptation to curriculum (EcS1). – Intra- and interdisciplinary connections (EcS2). – Social and labour usefulness (EcS3). – Didactic innovation (EcS4).

Source: Adapted from Ledezma, Breda, and Font (2024, p. 1062)

In the following sections, when we use the term DSC, we will be referring to the Didactic Suitability Criteria construct, meanwhile we will particularise the elements of this construct – *criteria*, *components*, and *indicators* – with their name or label, as appropriate.

METHODOLOGICAL ASPECTS

In this research, we followed a qualitative methodology from an interpretative paradigm (Cohen *et al*., 2018), which consists of a content analysis (Schreier, 2012).

This research was conducted in the Master's Programme in Teacher Training for Secondary and Baccalaureate Education (mathematics specialisation), taught by the public universities of Catalonia (Spain), during the 2019-2020, 2020-2021, and 2021-2022 academic years. Throughout their educational process, the prospective teachers take three modules in the master's programme: *Generic training*, *Complements of disciplinary training*, and *Educational internships*.

Within the *Complements of disciplinary training* module, a submodule of mathematical modelling is included. Prior to beginning this research, the first author observed the lessons of this submodule with the aim of knowing the addressed contents and the way in which they are taught. The submodule consists of four sessions (one per week) with a mainly expository methodology: in the first session, what is understood by modelling is explained and the cycle in Figure 1 is presented; during the second and third sessions, examples of modelling problems are presented, some of which the prospective teachers solve during both sessions; in the fourth session, the prospective teachers must expose the final task of the submodule, which

consists of presenting a modelling problem that includes its wording, solving, and curricular location of the mathematical content necessary for its solving.

Within the *Educational internships* module, the prospective teachers are assigned to internship centres under the supervision of a mentor teacher who must guide then in the educational institution. In such internship experiences, the prospective teachers must design and implement a teaching and learning sequence – known as *didactic unit* – which is determined by the educational institution, the student level, and the mathematical content taught in the time of the school year when they carry out their intervention in the classroom. Given these conditions, the prospective teachers cannot design a didactic unit exclusively focused on modelling, although they can implement modelling problems. When finishing their educational internship experiences, DSC are presented to the prospective teachers in the master's program so they use them in their MFPs and thus they can guide their reflection on their own practice.

Due to the health measures adopted because of the COVID-19 pandemic and its repercussions, the prospective teachers of the 2019-2020 course carried out their educational internship experiences in a virtual teaching context; those of the 2020-2021 course in a hybrid context; and those of the 2021-2022 course in a face-to-face context.

When finishing their educational process, the prospective teachers must prepare an MFP, which is an original, autonomous, and individual work which intends showing, in an integrated way, the knowledge and competences acquired during the master's programme. In addition, the MFP must contribute the prospective teachers to reflect on and delve into the analysis of their own practice, making it possible to propose elements for its improvement. For the preparation of the MFP, each prospective teacher is guided by a tutor professor from the master's programme who, in turn, supervised him/her during the educational internship experiences.

The basic structure of an MFP consists of five chapters: *Introduction*, where the didactic unit and the internship centre are contextualised; *Implementation analysis*, where the didactic suitability of the implemented unit is analysed using the DSC; *Redesign proposal*, where improvements to the didactic unit are proposed for a hypothetical redesign; *Competency self-assessment*, where each prospective teacher compares his/her starting competency level to that achieved when finishing his/her educational process in the master's programme; *Annexes*, where evidences of the implemented didactic unit can be included, the references cited in previous chapters, etc.

MFPs were the data source for this research; hence we performed a content analysis on 337 MFPs prepared by the prospective teachers during the three studied academic years: 122 from the 2019-2020 course, 117 from the 2020-2021 course, and 98 from the 2021-2022 course. To this end, we adapted the methodology used by Sánchez (2021) to the needs of this research, performing four *steps*.

In a *first step*, based on literature review and our knowledge, we prepared a list of keywords (and their derivations) related to modelling: context (contexts, contextualització, contextualitzar), model

(models, modelització, modelitzar), problema (problemes, problematització, problematitzar), and real (realitat, realista), which allowed us to identify the MFPs with references to modelling. In a *second step*, we recorded the data of each MFP (author, title, educational level, mathematical content), which allowed us to have an ordered database to consult the MFPs and record which ones included the keywords defined in the *first step*.

Due to a regularity found in the distribution of keywords in the *Implementation analysis* and *Redesign proposal* chapters of the MFPs, in a *third step*, we classified these documents in four levels of reference to modelling: at *Level 0*, we classified the MFPs that did not include references to modelling; at *Level 1*, we classified the MFPs that did not include working with modelling in the implemented didactic units (*Implementation analysis* chapter), but they did propose its inclusion in the redesign (*Redesign proposal* chapter); at *Level 2*, we classified the MFPs that included working with modelling in the implemented didactic units, but that did not propose improvements to enhance this process in the redesign; and at *Level 3*, we classified the MFPs that included working with modelling in the implemented didactic units and that proposed improvements to enhance this process in the redesign.

During this *third step*, once we established the four levels of reference to modelling described above, we conducted a triangulation in the following way: firstly, each author classified the MFPs according to these levels; secondly, we compared our classifications, achieving an agreement percentage between us of 96% (MFPs from the 2019-2020 course), 98% (MFPs from the 2020-2021 course), and 95% (MFPs from the 2021-2022 course); finally, we discussed the differences of classification and achieved a consensus, due to our experience in this type of analysis.

In a *fourth step*, we categorised the evaluative comments with references to modelling made by the prospective teachers, utilising the DSC *criteria* and *components* as aprioristic categories (Schreier, 2012). In this way, we could identify the aspects of the mathematical teaching and learning process that the prospective teachers prioritised in their reflections on the inclusion of modelling during their educational internship experiences. For this *fourth step*, we considered the MFPs classified at *Levels 2* and *3* described in the *third step*, since they contained references to modelling in the *Implementation analysis* chapter. Due to the consensus achieved by the authors during the *third step*, we performed this *fourth step* with no discrepancies, since it is an objective fact that the assessment of a certain DSC *component* in each MFP contains (or not) an evaluative comment related to modelling.

Box 2 exemplifies how we applied the four *steps* of content analysis to MFPs #005, #042 (both from the 2019-2020 course), #062 (from the 2020-2021 course), and #089 (from the 2021-2022 course). The choice of these four MFPs is justified by the fact that we classified them in different levels of reference to modelling.

Box 2 – Examples of content analysis with four MFPs

Content analysis	Analysed content
MFP #005 (2019-2020 course)	
First step	We identified the keywords "model" and "context".
Second step	It is a didactic proposal for teaching statistics in the third grade of secondary education (students aged 14-15).
Third step	The references of the identified keywords are related to the "educational model of the centre" and the "implementation context of the didactic unit". The redesign does not consider the inclusion of modelling. Therefore, we classified this MFP at *Level 0*.
Fourth step	-
MFP #042 (2019-2020 course)	
First step	We identified the keywords "modelling" and "problem".
Second step	It is a didactic proposal for teaching geometry in the first grade of secondary education (students aged 12-13).
Third step	We did not find evaluative comments related to modelling (using the DSC) in the *Implementation analysis* chapter. We found the following comment in the *Redesign proposal* chapter: "this would be a good problem for initiating students in modelling and can be proposed as a challenge for those students who successfully solved the first problem" (p. 24). Therefore, we classified this MFP at *Level 1*.
Fourth step	-
MFP #062 (2020-2021 course)	
First step	We identified the keywords "to model", "context", and "real".
Second step	It is a didactic proposal for teaching functions in the third grade of secondary education (students aged 14-15).
Third step	We found evaluative comments related to modelling in the assessment of the DSC in the *Implementation analysis* chapter. We did not find comments to improve working with modelling in the *Redesign proposal* chapter. Therefore, we classified this MFP at *Level 2*.
Fourth step	Among others, we found the following comment in the assessment of the EcS2 *component*: "the activities made required understanding and interpreting a problem in a real context, demonstrating the usefulness in daily life" (p. 24).
MFP #089 (2021-2022 course)	

Content analysis	Analysed content
First step	We identified the keywords "modelling" and "problem".
Second step	It is a didactic proposal for teaching functions in the fourth grade of secondary education (students aged 15-16).
Third step	We found evaluative comments related to modelling in the assessment of the DSC in the *Implementation analysis* chapter. We found a proposal for enhancing working with modelling in the *Redesign proposal* chapter. Therefore, we classified this MFP at *Level 3*.
Fourth step	Among others, we found the following comment in the assessment of the CS4 *component*: "This problem requires mathematical modelling to mathematise, analyse, and solve the situation posed in textual form" (p. 14). Also, we found the following comment in the assessment of the AS1 *component*: "Therefore, most of the activities and examples given in classroom were selected intending to show the applicability and utility of functions in daily life and their possible labour and/or academic future, trying to include the maximum number of possible aspects of application" (p. 19).

Source: Adapted from Ledezma, Breda, and Font (2024, p. 1068) and Ledezma, Sánchez, and Hidalgo-Moncada (2024, p. 13)

The content analysis methodology described above was perfectioned and validated throughout the time in which we conducted this research through individual analyses of MFPs prepared during the first two studied academic years (see Ledezma; Breda; Sánchez, 2021; Ledezma; Font; Sala, 2021).

RESULTS AND ANALYSES

In this section, we present the main results of this research from the content analysis performed on the 337 MFPs prepared by the prospective teachers during the three studied academic years.

From the search for keywords in the MFPs (*first step* of content analysis), we found terms related to modelling in 249 MFPs. After recording each MFP (*second step* of content analysis), we proceeded to classify them according to the levels of reference to modelling (*third step* of content analysis), and thus we obtained the results presented in Table 1.

Table 1 – Number of MFPs according to the levels of reference to modelling

Levels of reference	**Number of MFPs per academic year**		
	2019-2020	2020-2021	2021-2022
Level 0	36	30	22
Level 1	41	47	38
Level 2	21	24	25
Level 3	24	16	13
Total	122	117	98

Source: The authors

Table 1 offers an overview on how many MFPs reflected on the inclusion of modelling (*Levels 2* and *3*) and how many did not consider this process in their implemented didactic units (*Levels 0* and *1*). Thus, a first result to highlight is that there was a tendency in the three academic years regarding to the number of MFPs that implemented modelling in their didactic units: 45 in the 2019-2020 course (~37%), 40 in the 2020-2021 course (~34%), and 38 in the 2021-2022 course (~39%). In other words, around one third of the total number of MFPs prepared in each academic year included this process (123 of 337 MFPs).

Once we classified the MFPs in the four levels of reference to modelling (*third step* of content analysis), we proceeded to revise the MFPs that reflected on the inclusion of this process in their implemented didactic units. In this way, we continued the content analysis on the 123 MFPs classified at *Levels 2* and *3*, through the categorisation of the evaluative comments made by the prospective teachers in each DSC *component* that were related to modelling (*fourth step* of content analysis). Due to a lack of space, Table 2 only presents the number of evaluative comments that we found in the reflections on modelling according to each DSC (and not according to each *component*).

Table 2 – Number of evaluative comments according to the DSC

Criteria	**Number of evaluative comments per academic year**		
	2019-2020	2020-2021	2021-2022
Epistemic	54	48	12
Cognitive	19	20	2
Interactional	8	7	0
Mediational	8	9	1
Affective	40	24	8
Ecological	40	40	10

Criteria	Number of evaluative comments per academic year		
	2019-2020	2020-2021	2021-2022
Total	169	148	33

Source: The authors

To read Table 2, we must consider some methodological aspects assumed in this research. First, we considered as one evaluative comment to that statement that contained some (of the) keyword(s) determined in the *first step* of content analysis, but within a context related to modelling. Second, such comment had to be related to the reflections on one specific DSC *component*. Third, if the reflections on one DSC *component* contained more than one statement with these characteristics, we considered them together as one single comment. Fourth, the MFPs contained evaluative comments related to modelling in different DSC *components*, but not in all of them.

Table 2 also offers a more specific view on the DSC *criteria* – aspects of the mathematical teaching and learning process – that the prospective teachers prioritised in their reflections on the inclusion of modelling in their implemented didactic units. Thus, a second result to highlight is that there was a tendency in the three academic years regarding to the distribution of the evaluative comments on modelling among the different DSC *criteria*: on one hand, we made evident a high concentration in the *epistemic* and *ecological criteria*; on the other hand, there was a reduced number in the *interactional* and *mediational criteria*.

From the database generated in the *second step* – where one of the considered data was the mathematical content of each MFP according to the current curricular organisation (Departament d'Educació, 2019; Departament d'Ensenyament, 2008) – and the classification of the evaluative comments related to modelling that we made in the *third step*, Table 3 presents the mathematical contents addressed by the didactic units and the number of MFPs that implemented modelling according to these contents.

Table 3 – Number of MFPs that implemented modelling according to the mathematical contents

Mathematical contents	Number of MFPs per academic year		
	2019-2020	2020-2021	2021-2022
Algebra	4	8	10
Functions	19	10	8
Geometry	10	11	12
Numbers	2	2	3
Probability	1	2	1
Statistics	4	1	1

Mathematical contents	Number of MFPs per academic year		
	2019-2020	2020-2021	2021-2022
Trigonometry	5	6	3

Source: The authors

As mentioned in the Methodological Aspects section, among the determining factors to design and implement a didactic unit there are the student level and the time of the school year when the prospective teachers carry out their intervention in the classroom. In other words, both factors determined the mathematical content of the didactic units, and its choice did not depend on the prospective teachers. However, both factors did not determine whether modelling would be worked on or not. Regarding the time of the school year, in the context of this research, the educational internship experiences were carried out during six weeks from February to April of each academic year. Table 3 also shows a tendency in the three academic years regarding to the mathematical contents mostly used to implement modelling, where Functions and Geometry stood out; however, Probability and Statistics were the least used.

DISCUSSION AND CONCLUSIONS

In the previous section, we highlighted two results that we obtained from the content analysis performed on the 337 MFPs prepared during the three studied academic years, which we discuss in this section.

The first result to highlight of this research is that around one third of the total number of analysed MFPs implemented modelling in their didactic units (see *Levels 2* and *3* in Table 1). In general terms, there are limitations inherent to the educational internships period which the prospective teachers must face, both during the interventions in the classroom (work system of the internship centre, intervention of the mentor teacher, curricular times, etc.) and the requirements of the master's programme (time for the preparation of the MFP, feedback from the tutor professor, etc.). Although this programme includes a submodule dedicated to the teaching of modelling, the prospective teachers cannot design a didactic unit exclusively focused on this process. These reasons justify that modelling was neither the only mathematical process worked in their didactic units nor the only aspects on which they reflected in their MFPs. Taking into consideration these situations, in this research, we collected the evaluative comments made by the prospective teachers in their MFPs according to the importance they attributed to the inclusion of modelling during their educational internship experiences.

Regarding this first result, in which most of the prospective teachers did not include modelling during their educational internship experiences, we propose two plausible explanations. On one hand, in line with the position by Verschaffel *et al.* (2020), implementing modelling in mathematics lessons is not usual due to mainly the limitations of the educational context, student cognitive level, recommendations by the

mentor teacher, among other variables. On the other hand, the prospective teachers gave greater priority to other processes of mathematical activity, or they did not consider modelling as a relevant process within their implemented didactic units.

The second result to highlight of this research is that the distribution of the evaluative comments related to modelling was not equitable between the different DSC *criteria* (and *components*). In this sense, the *epistemic criterion* was that which concentred the greatest number of evaluative comments related to modelling, followed by the *ecological criterion*; meanwhile the *interactional* and *mediational criteria* were the least considered in these reflections. Due to a lack of space, in this article, we did not delve into the analysis of each DSC *component* in which we found evaluative comments related to modelling (see Ledezma; Breda; Font, 2024, for the MFPs from the 2019-2020 course; Ledezma; Sánchez; Hidalgo-Moncada, 2024, for the MFPs from the 2020-2021 course), however, we highlight some relevant *components* in this discussion.

Within the *epistemic criterion*, there is the ES3 *component*, which concentrated the greatest number of evaluative comments related to modelling. This is because, in this *component*, reflections are made on the mathematical processes worked in the implemented didactic units. Thus, the MFPs that included modelling, defined this process, and exemplified the tasks used to work on it. Regarding the latter, we highlight two particularities: on one hand, definitions of modelling varied, from considering this process as the mathematisation (translation into mathematical language) of a problem-situation, to as a cyclical process made up of phases; on the other hand, the tasks used to work on modelling did not always meet the characteristics of this type of problems (see Ledezma; Font; Sala-Sebastià, 2024). These particularities led us to question whether all the prospective teachers actually implemented modelling from what they learnt in the master's programme or they only tried to get close to this process despite of the limitations inherent to the educational internships period (see a particularised analysis in Ledezma; Sol *et al*., 2022).

Within the *ecological criterion*, there are the EcS2 and EcS3 *components*, which also included several evaluative comments related to modelling. Although, in some cases, the prospective teachers overlapped their reflections in both *components*, we highlight that these assessments considered modelling as a tool to relate the contents addressed in the didactic units to extra-mathematical contexts and from other disciplines (especially from physics and economics). In the same way, Functions was the most used mathematical content by the prospective teachers to work on modelling, in a position coinciding with Michelsen's (2006), who considers this content as a useful tool for developing modelling in the classroom.

Resuming main question that guided this research, on what aspects of the mathematical teaching and learning process prospective teachers prioritised in their reflections on the inclusion of modelling during their educational internship experiences, we can affirm that they prioritised the *epistemic* and *ecological* aspects (*criteria*).

Although up to this point, we have discussed the results that allowed us to answer the main question that guided this research, it is interesting to delve into the

influence of the three teaching contexts on the reflections made by the prospective teachers on the inclusion of modelling in their implemented didactic units. To address this point, we consider two additional aspects that the results of this research suggest.

The first aspect is that around two thirds of the total number of analysed MFPs (214 of 337 MFPs) did not implement modelling in their didactic units (see *Levels 0* and *1* in Table 1). We discard as an explanation that the prospective teachers did not have knowledge about modelling and its inclusion in a mathematical teaching and learning process, since the master's programme in which this research is contextualised dedicates a submodule exclusively to teaching this process. We also discard as an explanation that the COVID-19 pandemic has been a context that did not foster modelling, since the media included information that allowed to design modelling tasks/problems, as well as models to represent the evolution of the pandemic, which provided modelling with a relevant social value. Nevertheless, a plausible explanation is that, in terms of the DSC, a teacher must ensure to *a priori* that these *criteria* are met as much as possible; however, the implementation context forces him/her to make decisions on what aspects he/she should prioritise, relegate to the background, or simply omit. In the case of this research, although one of the *indicators* of the ES3 *component* gives importance to the development of, among others, the modelling process in the classroom, the influence of the teaching contexts made these prospective teachers to prioritise other aspects of the mathematical teaching and learning process such as, for example, the work system of the internship centre (recovering contents after lockdown periods, closeness to the work methodology), dynamics of the classroom, other relevant processes of mathematical activity, etc.

The second aspect requires considering the scarce evaluative comments related to modelling in the *components* of the *interactional* and *mediational criteria* as the clearest evidence. In the first two academic years, the prospective teachers commented on the difficulties to develop group tasks with the students, as suggested by Shahbari and Tabach (2019) for working with modelling: while in the 2019-2020 course the impediments were the virtual teaching contexts and the scarce interaction between the students, in the 2020-2021 course the impediments were the health measures of social distancing and reduced capacity in the classrooms. Although both situations affected the *interactional* aspect of the mathematical teaching and learning process, there were positive assessments in the IS3 *component* given the dynamics of individual work for modelling tasks. On their part, the evaluative comments related to modelling found in the *mediational criterion* were focused, almost entirely, in the assessment of the MS1 *component*, in which the prospective teachers highlighted the use of dynamic-geometry software for working with modelling. However, the reflections made by the prospective teachers only attributed them a complementary role for this process and not as a mean for the teaching and learning of modelling.

The results of this research have different implications. Firstly, by offering an overview on the teaching of modelling in the master's programme in which this study is contextualised, these results allow us to

identify the aspects to improve in the submodule dedicated to this process and in the feedback made by the tutor professors to the prospective teachers in their MFPs. For example, it would be interesting to consider the recommendations of other successful educational experiences in modelling to add them to this master's programme (see Borromeo Ferri, 2018; Greefrath *et al*., 2022; among others). Secondly, these results are a detailed sample of the use of the DSC as a tool for reflecting on one particular aspect such as modelling is in a context of teacher education. Finally, these results allow us to lay the foundation for a guideline of DSC exclusively focused on mathematical teaching and learning processes that include modelling in the classroom (see an advance in Ledezma; Font *et al*., 2022).

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DATA AVAILABILITY STATEMENT

MFPs are public documents, but they are not available on the internet, but they are kept in the Autonomous University of Barcelona (Plaza Cívica, Campus de la UAB, 08193, Sardañola del Vallés, Barcelona, Spain). If someone wants to revise the MFPs analysed in this study, he/she can ask for it to the authors.

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