An Investigation of Primary School Arithmetic in the United States between Colburn and Thorndike

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Abstract

The following study provides a brief account of the development of US American didactics of arithmetic between 1821 and 1917, as it appears in the textbooks of the time and with special reference to Warren Colburn (1793-1833) and Edward Lee Thorndike (1874-1949). The core of the study is the analysis of a forgotten, yet in its spirit still actual debate between Dewey and Phillips concerning the nature of arithmetical knowledge. My discussion owes a great deal to the remarkable historical work of Robert G. Clason.

Key Words: Arithmetic, Semiotics; Coburn, Dewey, Grube, Dewey, Graßmann, McLellan, Pestalozzi, Phillips, Schubert, Thorndike.

The most important objective of this paper is to show that the multitude of approaches to the teaching of primary arithmetic proposed in the US American textbooks published between 1821 and 1917 as well as the general pedagogical and philosophical debates accompanying them have a common focus. This common focus surfaces most evidently in the debate between J. Dewey and D. E. Phillips. While both authors emphasize the *relational* nature of the concept of natural number, they differ in their assessment of the type of relationship involved. Phillips regards counting and the formal number series as the foundation of the number concept and of numerical experience. Dewey, on the other hand, takes measurement and the concept of ratio as the foundation of numerical experience.

Before 1821 arithmetic was given no significant place in American education. Monroe writes that prior to 1821 the study of arithmetic consisted only in a brief introduction to the so-called "commercial arithmetic" (cf. Monroe in Clason 1968 50 f.). F. Cajori characterizes US American arithmetic textbooks published before 1820 as „Pandora's boxes of ill-formed rules to be committed to memory" (Cajori 1890, 49). While it is true that the number of US American universities, demanding knowledge of arithmetic as a condition for admission rose during the second half of the 18th century, hardly anyone recommended that arithmetic should be taught to young children in primary schools and nobody saw the teaching of arithmetic as a

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fundamental ingredient in reaching an adequate general education (Allgemeinbildung). Warren Colburn‘s „First Lessons in Arithmetic on the Plan of Pestalozzi“, a book published in 1821, counts as a turning point in the history of American didactics of arithmetic.

Colburn was one of the first and most influential advocates of the idea of providing arithmetic instruction to all young children. This position was supported by the idea that early age arithmetic instruction naturally belongs to general education (Allgemeinbildung). Colburns “First Lessons” has known 44 editions between 1821 and 1917. Before Colburn the general view was that arithmetic should be taught beginning with the age of eleven. Colburn demanded that arithmetic teaching should begin at the age of five or six. This shift was intertwined with other changes concerning affecting the general understanding of arithmetic. Before 1821, the main reasons to study arithmetic were either practical (for instance, in order to be able to come to terms with commercial problems) or determined by the view that arithmetic is an important conversation subject between educated people. The idea that arithmetic has something to do with general education gained ground due, for instance, to the growing influence of the pedagogical and philosophical ideas of Pestalozzi. (Cf. Clason 1968, 57 ff.). Clason characterizes Colburn’s arithmetic as:

> "(…) an example of a mental or intellectual arithmetic, that is, problems are to be solved mentally (...) rather than through formal rules or principles, however established. In such books 3/8 of 32 would be analyzed orally as 3 1/8‘s of 32 or 3 fours, rather than as an instance to which a rule for multiplying applies.“ (Clason 1968, 59)

Colburn’s arithmetic contains neither definitions, nor general principles or rules. It appears as a classical example of an approach emphasizing mental exercise and verbalization of more or less ad hoc developed solution strategies for individual arithmetic problems.

Clason’s discussion of a large number of textbooks makes it possible to recognize a new trend in the design of arithmetic textbooks characterized by a move away from textbooks written exclusively for teachers, towards textbooks to be used independently by the students. The textbook gradually replaces the direct teacher-student dialog. It becomes the most important medium of learning. This development reaches a new level in Thorndike’s behaviorist conception of mathematics teaching, which is later on refined by B. Skinner and others.

Contrasting positions such as those of Brooks, on the one hand, and Mansel, on the other, are characteristic for this period. In his „*Prolegomena Logica: An Inquiry into the Psychological Character of Logical Process*“ of 1860 Henry L. Mansel affirmed, that arithmetical concepts were not part of a logical structure but represent directly perceivable facts or states of affairs. Thus, arithmetic appears as a collection of practical and logically independent methods and algorithms, generated in the confrontation between the mind and different specific (mostly practical) problem situations. From this point of view, any logically oriented re-structuring of arithmetic appears as a betrayal of the original unity of thought and experience. We encounter here a kind of psychological realism with respect to the arithmetical procedures. If the arithmetical procedures (basically algorithms) can only grow out of the direct clash between the mind and a given problem situation, the decisive ingredients of the teaching/learning process are not textbooks or some formal presentation of the procedures, but the actual effort of the student (eventually guided by a teacher) of coping with experience. In Mansel’s case, this plausible position is linked to a radical, and I would say rather short sited, rejection of taking the logical structure of arithmetic understood as a coherent logical system into account. This attitude downplays the role of representations by means of artificial sign systems in the teaching/learning of arithmetic.

In a series of books published beginning with 1859, Edward Brooks expressed the view that Mansel's conception of arithmetic would make it impossible to establish arithmetic as a "science of number" (rather than seeing it as a loose collection of useful algorithms). Brooks saw, in contrast, arithmetic as a logical system. Some of the arithmetic textbooks written during this period showed a great interest for organizing the various arithmetical procedures into a coherent whole. As we shall see, however, the dominant position remained Mansel’s. The tension between these two poles (arithmetic as a logically organized system versus arithmetic as a collection of practical computation procedures) is present throughout the entire period discussed here. It is a time in which the didactic and philosophical reflections concerning the foundations of mathematics gained momentum influencing each other a great deal. The arithmetic textbooks of this time became the battleground for the various conceptions concerning the foundations of arithmetic on the one hand, and the debates about the nature of knowledge on the other hand. The entire period is characterized by the search for a satisfactory mathematical formulation of the concept of natural number and by the requirement of teaching arithmetic in school accordingly. Characteristic for this mixture of mathematical, philosophical, psychological, and didactic ideas are the works of August Wilhelm Grube, James A. McLelland, A. F. Ames and John Dewey, William W. Speer and finally D. E. Phillips (compare Clason 1968 215 f.; 229).

Grube was considered as a follower of Pestalozzi. His ideas about the didactics of arithmetic can be found in his "Guide to the arithmetic in the elementary school" of 1842, which appeared in English translation in 1878 and 1891. According to Grube mathematical knowledge has its origin in immediate observation, so that one must always proceed from the concrete to the abstract. This "concrete", as a rule, must be encountered in the world of sense objects. Essential is Grube’s emphasis on the cardinal number concept and, related to that, an associated number-atomism as well as a strict rejection of the concept of ordinal number. Grube writes:

> „Suppose that the number four is to be taught. Four is so many (things), not a sign which we call a figure; so to give an idea of four, the teacher must present things, not a sign. Four has an individuality of its own, and is worthy of introduction without the aid of its smaller sisters; so do not let the children arrive at four by counting. There is no call at any time for any teaching of counting by ones. The pupil learns the right succession of numbers unconsciously as numbers are presented to him in a logical order” (Grube in Clason 1986, 200).

Grube's ideas found their way into Wentworth and Reed's "The First Steps in Number" of 1891, which had a huge impact on US American mathematics education. The chapter dedicated to the number "8" in this book has no less than 34 pages! It is noteworthy that, in accordance with Grube’s views, "counting" and ideas like "one more" or "one less" are nowhere used in this book. Moreover, the numbers are not even introduced in the usual order, but rather as follows: 1, 4, 7, 9, 6, 5, 2, 3, 8 (compare Clason 1986 199 f.).

McLellan and Dewey allow counting, but measurement remains the basis for their introduction of number. They describe their approach as follows:

> „We began with a vague whole, an undefined unity; we broke it into parts (analysis) and by relating (counting) the parts we arrived at our unity again; the same unity, yet not the same as regards the attitude of the mind towards it. (...) This rhythmic process of parting and joining leads to all definite quantitative ideas.” (McLellan & Dewey in Clason 1968, 207).
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> „The unit is never to be taught as a fixed thing (e.g., as in the Grube method), but always as a unit of measurement. *One* is never one thing simply, but always that one thing used as a basis for counting off and thus measuring some whole or quantity” (McLellan & Dewey in Clason 1968, 209).

The focus therefore is on the emphasis of a holistic experience, as the basis and starting point for measurement and for the development of the number concept.

Characteristic of Speer’s approach is his one-sided emphasis of the relational idea. Clason writes: „The basic notion of number is directly-perceived relation between quantities“, and further: „Speer asserts that quantity is not perceived without relation“ (Clason 1968, 136). Speer himself writes:

> „The fundamental thing is to induce judgment of relative magnitude. The presentation regards the fact that it is the *relation* between things, which makes them what they are. The *one* of mathematics is not an individual, separated from all else, but the union of two like impressions: the *relation* of two equal magnitudes. A child does not perceive this one until he sees the equality of two magnitudes” (Speer in Clason 1968, 138).

Speer considers „ratio” as a „directly perceived quantitative relation” (Clason 1968, 138). That leads him to reject the use of discrete entities for the initial teaching of arithmetic:

„By presenting divided magnitudes (...) we destroy the wholes we wish compared, and call upon the child for a synthesis for which he is not prepared. The problem does not require him to make a comparison of the magnitudes, but merely to count the how many. We force upon the attention isolated units and operations for which the mind has no need, and which, by being thus pushed into the foreground, tend only to intellectual chaos" (Speer in Clason 1968, 210).

It is symptomatic for these debates that issue of representation is hardly ever addressed. This is not really surprising, because as long as each mathematical concept is seen as expressing a relationship and if further every relationship is seen as directly perceived (e.g. while working with continuous magnitudes), then the question of the ways and means of representation appears as secondary. Abstract mathematical signs are regarded as mere symbols used to record familiar relations between concrete magnitudes. Indeed Speer writes that "operations on symbols are barren without the experience which gives them significance" (Speer in Clason 1968, 136). This is not as harmless as it perhaps may seem. Commenting on Speers work Clason writes: "Symbols for operations are almost totally absent from this series. It is only after a considerable search that I find even a plus sign on page 150 of the second text of the series" (Clason 1968, 136).

The period from 1885 until 1917 is described by Clason as one dominated by the idea that in mathematics one does not operate with things given in sense experience but rather with gradual regarded the growing emphasis on the idea that you cannot operate in mathematics abstractions designated by symbols. This view is reflected in the US American arithmetic books of this period, but this does not, as one might perhaps suspect, find its way into the arena of arithmetic teaching. At the level of arithmetic textbooks the emphasis on abstract notions and symbols has a surprising effect: Many of the books and book-series which are designed to be used in teaching arithmetic at the primary level, the initial number-concepts which are usually taught during the first two school years are not treated at all. In those cases in which the initial number-concepts are treated in these books, this is done only in those sections intended for the last part of the second school-year. (Cf. Clason 1968, 222)

One encounters here a tension between a treatment of the number-concept, which is seeking to uncover something like its very "essence", something supposed to be more fundamental than the common mathematical symbolic practice, an experience which can only be made and described in terms of our dealings with sensible objects, on the one hand, and a mathematical symbolic practice, on the other hand, which is only relevant for other "higher" areas of mathematics. The latter is often banned from early teaching.

This separation is even more accentuated in the debate opposing Dewey and Phillips and in a number of other writings in which Dewey argues against the rising empiricism of the time. Particularly relevant for Dewey position is his attack on empiricism from 1884, in which he anticipates Quine's "Two Dogmas of Empiricism". Characteristic of the procedure of the empiricists is, according to Dewey to model continuous reality in terms of some arbitrary categories that are adopted a priori:

> “This tendency has nowhere been stronger than in those who proclaim that ‚experience‘ is the sole source of all knowledge. They emasculated experience till their logical conceptions could deal with it; they sheared it down till it would fit their logical boxes; they pruned it till it presented a trimmed tameness which would shock none of their laws; they preyed upon its vitality till it would go into the coffin of their abstractions.“ (Dewey in Clason 1968, 306)

Dewey is looking for a "pragmatic" basis as an alternative to static empiricism, one that puts the context of action, connecting the human mind and the world in the foreground. This view is reflected particularly clearly in the Dewey Phillips debate on the number concept and the design of the arithmetic instruction. Both Phillips and Dewey try to uncover the "essence" of numerical experience, but their approaches are very different.

Phillips appears to be influenced by Hermann Schubert’s account of the concept of natural number, which had been published in the *The Monist* (“Notion and Definition of Number”, and “Monism in Arithmetic”). Schubert's position in turn is linked to various early German contributions to the foundations of arithmetic developed during the 19th century (compare, for instance Radu 2000). One of the most significant contributions of this kind was Hermann Graßmann’s “Lehrbuch der Arithmetik” from 1861. This book contains the first almost accomplished axiomatic treatment of the natural number concept. Graßmann regards the ordinal number concept as the epistemological and mathematical essence of arithmetic and his book was intended and used by him for teaching the natural numbers in school. Phillips invokes a number of anthropological and psychological studies in support of his idea that the notion of ordinal number arises independently of the cardinal number concept. Phillips writes:

> „When a child sits rhythmically tapping his foot, saying, ‚one, two, three’, etc. even to one hundred, does he have at first a vague idea of muchness to be measured, or when done, the slightest idea of the somewhat counted? Now in all such counting the process has no conscious meaning whatever save that each number is followed by this and preceded by that. The same is true for adults after a certain limit is reached” (Phillips 1897-1898, 593).

Further on we read:

> „This series idea, established by a multitude of successive and rhythmical sensations conveyed through the different senses, ceases to be symbolized by different touch sensations, circulation, breathing, and movements, by sounds, clock-strokes, etc., by varying objects in the field of vision, and becomes a general idea applicable to any series of successions.“ (Phillips 1897-98, 229)

But the most significant passages – the view expressed in them is criticized by Thorndike in his treatment of arithmetic under the heading "The series idea overemphasized" (compare Thorndike 1922, 4) - is the following one:

> „This is essentially the counting period, and any words that can be arranged into a series furnish all that is necessary. Counting is fundamental, and counting that is spontaneous, free from sensible observation, and from the strain of reason. A study of these original methods shows that multiplication was developed out of counting, and not from addition as nearly all textbooks treat it. Multiplication is counting. When children count by 4's, etc. they accent the same as counting gymnastics or music. (...)

Phillips regards the ordinal concept of natural number as the foundation of the entire teaching of arithmetic. This foundation, however, the spontaneous counting activity of everyday life combined with a cultural artifact: "a systematic series of names" which are "first learned and applied to objects later" (Phillips 1897-1898, 238).

Dewey characterizes Phillips' conceptions as follows:

> „This theory, as I understand it, separates the counting process, in which number undoubtedly has its origin and development, from the consciousness of quantity, holding that the number idea in the form of a series, is a distinct process psychologically and pedagogically from the ratio idea (...)" (Dewey 1897-1898, 427).

Here Dewey rejects Phillips' strong emphasis on the "systematic series of names" as based on a confusion between spontaneous counting-games in everyday contexts, on the one hand, and the mechanical recitation of an arbitrary series of names accompanied by gestures, on the other. Such a recitation is „no more counting than is the saying off, or counting out rhymes like: ‚Eeny, meney, mony, mi,' etc." (Dewey 1897-1898, 427). Philipps replies by pointing out that Dewey and McLellan's idea of number was inspired by Hegel's "idea of number as ratio" (Phillips 1879-1898, 256). In the end, Phillips summarizes "the fundamental difference" between his own conception and that of Dewey and McLellan as follows:

> „Prof. Dewey (...) contends that counting begins with a vague whole to be made definite, to which a unit is applied, and the *how many* is recognized as a collection of units marking off the somewhat counted. Later comes the period of spontaneous counting as an end in itself. After this period, he would have a second application to things for the purpose of emphasizing the measuring or ratio idea. On the other hand, the evidence points to the fact that the idea of succession is unconsciously developed in the child by the never ending recurrence of various similar sensations, that later the force of these processes coming to consciousness manifests itself in *the series idea*. (...) In this sense, *counting precedes the application of the series to things*. Thus, the period of spontaneous counting precedes the period of counting in the sense of recognizing a quantity as a collection of units" (Phillips 1897-1898, 597. Emphasis added).

Based on a primitive natural habit, humans develop an extensive ordered system of signs (the number series) that requires nothing except this habit. Thus, the elementary arithmetic appears as the product of a spontaneous disposition of our mind combined with a formal tool represented by the usual series of counting words and symbols. Only the applications and further development of this tool in measurement and in higher mathematics make a process of reflection of the kind emphasized by Dewey necessary. According to Phillips it is only in these more advanced contexts that the idea of number as a ratio between magnitudes constantly emphasized by Dewey is meaningful. The teaching of arithmetic in primary school should therefore focus on the abstract number series and on using it as a basis for the development of abstract "computing skills" rather than on measurement. Dewey rejects this approach:

> „Independence from any specified application is readily taken to be equivalent to independence from application as such; it is as if specialists, engaged in perfecting tools and having no concern with their use and so interested in the operation of perfecting that they carry results beyond any existing possibilities of use, were to argue that therefore they are dealing with an independent realm having no connection with tools or utilities. This fallacy is especially easy to fall into on the part of intellectual specialists. (...) Those who handle ideas through symbols as if they were things (...) and trace their mutual relations in all kinds of intricate and unexpected relationships, are readily victims of thinking of these objects as if they had no sort of reference to things, to existence." (Dewey 1929, 154)

This type of argument is the expression of Dewey's sharp distinction between a *practice in the material world*, on the one hand, and an *abstract symbolic practice*, on the other. The latter he describes as a study of the merely symbolic tool for the sake of the tool and rejects it.

Thorndike's *Arithmetic* of 1917 was an attempt to end these "sectarian" debates concerning the true "essence" of number. Only observable and measurable associations between certain stimuli and certain responses are taken into account. In the first chapter of his *Arithmetic*, Thorndike identifies four meanings of the number concept: "series meaning", "collection meaning", "ratio meaning", and „relational meaning". Summarizing the debates of his time he writes:

> „Ordinary school practice has commonly accepted the second meaning as that which it is the task of the school to teach beginners, but each of the other meanings has been alleged to be the essential one – the series Idea by Phillips, the ration idea by McLellan and Dewey and Speer, and the relational idea by Grube and his followers."

He than adds:

> „It should be obvious that all four meanings have claims upon the attention of the elementary school. Four is the thing between three and five in the number series; it is the name for a certain sized collection of discrete objects; it is also the name for a continuous magnitude equal to four units (...); it is also, if we know it well, the thing got by adding one to three or subtracting six from ten or taking two two's or half eight. To know the meaning of a number means to know somewhat about it in all of these respects. The difficulty has been the narrow vision of the extremists" (Thorndike 1922, 7).

Thorndike's *Arithmetic* emphasizes the need to anchor number names in different contexts, regardless of questions about the nature of either thought or abstraction or the numbers themselves. Learning is "essentially the formation of connections or bonds between situations and responses", that is, the formation of habits. By pointing out that "habit rules in the realm of thought as truly and fully as in the realm of action" (Thorndike 1922, v) debates such as that between Phillips and Dewey are pushed aside and so is the idea that the teaching of arithmetic should be based on some logically articulated theory of natural number.

Given the confusing debates about the essence of number, Thorndike’s approach is refreshing and seems quite reasonable. Thorndike's conception, however, has shortcomings of its own. His conception is guided by a strict economy. The outcome is a restrictive conception of arithmetic and particularly of problem solving. Thorndike admits only arithmetical problems that are relevant for everyday life. Learning means inducing and reinforcing useful habits. Its objective is not the acquisition of knowledge or of skills, but rather “working knowledge”. Thorndike dismisses the efforts to organize arithmetic into a coherent logical system as irrelevant for teaching purposes. Other types of problems are dismissed as catch-problems. A catch-problem forces the learner to operate against “some customary habit”. Thorndike defines of “catchiness” as “contra-previous-habitness” (Thorndike 1922, 21). Among Thorndike’s examples of catch-problems we find, for instance, “What must you add to 19 to get 30?” (ibid. 21). We are told that such problems are of limited use, for abstract reasoning is beyond the scope of arithmetic teaching. Reasoning “should be chiefly a force organizing habits, not opposing them“ (ibid. 22). Thorndike mentions another form of reasoning needed to solve such problems for which “no ready-made habitual response is available (ibid. 185). In his *Psychology of Arithmetic*, however, this type of reasoning is used only for selecting one of two available habits.

Thorndike's conception does not leave much room for complex problem solving, let alone for problem posing and generalization. His focus on the economy eventually culminates in the demand characteristic "programmed instruction" according to which a good program must reduce possible mistakes to a minimum. Even though Thorndike carefully avoids ideological debates about the “true” psychological nature of the number concept, there is one issue on which he takes a position which, with the exception of Phillips, is endorsed by all the authors discussed in this paper: „A child must not be left interminably counting; in fact the one-more-ness of the number series can almost be had as a by-product” (Thorndike 1922, 6).

One important feature of today’s debates in mathematics education is the opposition between two positions. The first position emphasizes *conceptual understanding* and *higher order thinking skills* whereas the second insists on the fundamental importance of *basic (honest mathematical) knowledge*. The previous discussion suggests that even though they belong to a different time Phillips, Dewey, McLellan, Thorndike and all the others are our contemporaries.

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